LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2018**

MT 3503– VECTOR ANALYSIS & ORDINARY DIFF. EQUATIONS

Date: 31-10-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Section – A

ANSWER ALL THE QUESTIONS: (10x2=20 Marks)

1) Find the directional derivation of $\emptyset = x + xy^2 + yz^3$ at (0, 1, 1) in the direction of the vector $2\vec{i} + 2\vec{j} - \vec{k}$

2) Find the equation of the tangent plane to the surface $x^2 + 2y^2 + 3z^2 = 6$ at the point (1, -1, 1).

3) If $F = 3xy\vec{i} - y^2\vec{j}$ and C is the curve x =t, y = 2t^2 from (0, 0) to (1, 2) find $\int_c F.dr$

4) What is the necessary and sufficient condition for line integral to be independent of path of integration.

5) State stokes theorem.

6) Show that for a closed surface S enclosing a region of volume V.

$$\iint_s \left(ax\vec{i} + by\,\vec{j} + cz\,\vec{k}\right).ds = (a + b + c)v$$

7) Solve $p^2 - 5p + 6 = 0$.

8) Solve $y = (x-a)\,p - p^2$

9) Find the particular integral of $(D^2 - 3D + 2)\,y = \sin 3x$.

10) Solve $(D^2 - 5D+6)\,y = 0$.

Section – B

ANSWER ANY FIVE QUESTIONS: (5x8=40 Marks)

11) If $\nabla\emptyset = (y + y^2 + z^2)\vec{i} + (x + z + 2xy)\vec{j} + (y + 2zx)\vec{k}$ and if $\emptyset\,(1,1,1) = 3. Find\ \emptyset$.

12) Show that $\nabla^2 r^n = n\,(n+1)r^{n-2}$

13) Find the value of the integral $\int_c A.dr\ where\ A = yz\vec{i} + zx\vec{j} - xy\,\vec{k}\ if$

 a) C is the curve defined by x =t, y =t^2, z=t^3 drawn from O (0, 0, 0) to Q (2, 4, 8)

 b) C is the straight line joining (0, 0, 0) to (2, 4, 8).

14) Evaluate $\iint A.n\,d.s$ if $A = 4y\vec{i} + 18z\vec{j} - x\vec{k}$ and S is the surface of the portion of the plane 3x + 2y + 6z = 6 contained in the first octant.

15) Verify Green's theorem in the plane for the $\int_c x^2y\,dx + ydy$ where C is the curve enclosing the region R bounded by the line y =x and the parabola $y^2 = x$.

16) Solve $\frac{dy}{dx} - ytanx = \frac{sinx\,cos^2x}{y^2}$.

17) Solve $(D^2 + 16)y = 2e^{-3x}$.

18) Solve by variation of parameters $\frac{d^2y}{dx^2} + y = \sec x$

Section – C

ANSWER ANY TWO QUESTIONS: (2x20= 40 Marks)

19) a) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

Also show that if $\nabla^2 f(r) = 0$ *then* $f(r) = \frac{\alpha}{r} + \beta$ where α and β are arbitrary constants.

b) Find the value of a if

$A = (axy - z^2)\vec{i} + (x^2 + 2yz)\vec{j} + (y^2 - axz)\vec{k}$ is irrotational.

20) Verify the divergence theorem for $A = (x + y)\vec{i} + x\vec{j} + z\vec{k}$ taken over the region V of the cube bounded by the planes x=0, x=1, y=0, y=1, z=0, z=1.

21) a) Solve $xp^2 - 2yp + x = 0$.

b) Solve $p^2 + 2yp \cot x = y^2$.

22) a) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x$

b) Solve $[D^2 + 4D + 5]\, y = e^x + x^3 + \text{Cos } 2x$.

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