LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2018**

MT 5505– REAL ANALYSIS

Date: 27-10-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART-A

Answer all the questions: $(10 \times 2 = 20)$

1. Define similar sets and give an example.

2. Prove that N is not bounded above.

3. Define metric space.

4. Define Cauchy sequence.

5. Define homeomorphism.

6. Define uniformly continuous.

7. State Rolle 's Theorem.

8. Define local maximum and local minimum of a function at a point.

9. Define bounded variation.

10. Define Riemann-Stieltjes integral of a function f with respect to α in $[a,b]$.

PART-B

Answer any FIVE questions: $(5 \times 8 = 40)$

11. State and prove Cauchy-Schwarz inequality.

12. Show that e is irrational.

13. Let Y be a subspace of a metric space (X,d). Then prove that a subset A of Y is open in Y if and only if $A = Y \cap G$ for some set G open in X.

14. Prove that the continuous image of a compact metric space is compact.

15. Let $f:(X,d_1)\to(Y,d_2)$ be uniformly continuous on *X*. If $\{x_n\}$ is a Cauchy sequence in *X*, prove that $\{f(x_n)\}$ is a Cauchy sequence in *Y*.

16. State and prove intermediate value theorem for derivatives.

17. If $f\in R(\alpha)$ on $[a,b]$ and $f\in R(\beta)$ on $[a,b]$, then for any pair of constants λ and μ, then prove that the following are true. (i) $f\in R(\lambda\alpha+\mu\beta)$ on $[a,b]$.

(ii) $\int_a^b f\,d(\lambda\alpha+\mu\beta)=\lambda\int_a^b f\,d\alpha+\mu\int_a^b f\,d\beta$

18. State and prove the formula for integration by parts.

PART-C

Answer any TWO questions: $(2\times 20=40)$

19. (a) Prove that every subset of a countable set is countable.

(b) Prove that the set $\Re$ is uncountable. **(10+10)**

20. Prove that let (X,d) be a metric space, ***G*** be the collection of all open sets in *X* and ***F*** be the collection of all closed sets in *X*. Then ***G*** is closed with respect to arbitrary unions and finite intersections and ***F*** is closed with respect to arbitrary intersections and finite unions.

21. State and prove Bolzano theorem and deduce Intermediate value theorem.

22. (a) State and prove Taylor's theorem.

(b) Let $f\in R(\alpha)$ on $[a,b]$, α be differentiable on $[a,b]$ and α' be continuous on $[a,b]$.

Show that the Riemann integral $\int_a^b f\,\alpha'\,dx$ exists and $\int_a^b f\,d\alpha=\int_a^b f\,\alpha'\,dx$. **(10+10)**

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