LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2019**

16/17UMT5MC03 – LINEAR ALGEBRA

Date: 02-11-2019 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART-A

Answer All Questions **(10 x 2 = 20)**

1. If V is a vector space over F, prove that

   (i) $(-\sim)v = -(\sim v)$.

   (ii) $0v = 0$ for $\sim \in F$ and $v \in V$.

2. If F is the field of Real numbers, prove that the vectors $(1,1,0,0)$, $(0,1,-1,0),(0,0,0,3)$ in $F^{(4)}$ are linearly independent.

3. In an inner product space V over F, show that $\|\sim u\| = |\sim|\|u\|$ for $\sim \in F$ and $u \in V$.

4. Define norm of a vector in an inner product space.

5. Show that $T : F^n \to F$ given by $T(\sim_1,\sim_2,\sim_3,...,\sim_n) = \sim_1$ is a linear transformation.

6. Define rank of a linear transformation.

7. Define characteristic root of a linear transformation.

8. Define invariant subspace.

9. Examine whether the matrix $\begin{pmatrix} 0 & -a+ib \\ a+ib & 0 \end{pmatrix}$ is skew Hermitian or not.

10. When is a linear transformation said to be unitary?

PART- B

Answer Any FIVE Questions **(5x8 = 40)**

11. Show that $V = F^{(n)}$ where F is field, is a vector space over F.

12. Let S and T be subspaces of V. Prove that

    (a) $L(S)$ is a subspace of V.

    (b) $L(S \cup T) = L(S) + L(T)$ (4+4)

13. Write down Schwarz inequality and establish the same.

14. If V is finite dimensional over F, prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.

15. If ª $_1$, ª $_2$ … ª $_k$ in F are distinct characteristic roots of $T \in A(V)$ and if $v_1,v_2…,v_k$ are characteristic vectors of T belonging to ª $_1$, ª $_2$ … ª $_k$ respectively. Prove that $v_1,v_2…,v_k$ are linearly independent over F.

16. If F is a field, V the set of polynomials in x of degree $n-1$ or less over F and D an operator on V is defined by $({}_0 + {}_1x + {}_2x^2 + {}_3x^3 + \ldots + {}_{n-1}x^{n-1})D = {}_1 + 2\,{}_2x + 3\,{}_3x^2 + \ldots + (n-1)\,{}_{n-1}x^{n-2}$. Find a matrix of D in the basis $\{1, x, x^2, x^3 \ldots, x^{n-1}\}$.

17. If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, prove that $T = 0$.

18. (a) If $T \in A(V)$ is Hermitian then prove that all its characteristic roots are real.

(b) Prove that $T \in A(V)$ is unitary if and only if $TT^* = 1$. (4+4)

PART- C

Answer any TWO Questions **(2x20 = 40)**

19. (a) If V is finite dimensional and if W is a subspace of V, prove that V is finite dimensional, $\dim W \leq \dim V$ and $\dim V/W = \dim V - \dim W$.

(b) State and prove Gram Schmidst orthogonalisation process. **(10+10)**

20. (a) If A is an algebra with unit element over F, prove that A is isomorphic to a subalgebra of $A(V)$ for some vector space V over F.

(b) If V is finite dimensional over F, prove that $T \in A(V)$ is regular if and only if T maps V onto V.

(10+10)

21. (a) If V is n-dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $v_1, v_2, v_3, \ldots, v_n$ and the matrix $m_2(T)$ in the basis $w_1, w_2, w_3, \ldots, w_n$ of V over F, prove that there is an element $C \in F_n$ such that $m_2(T) = Cm_1(T)C^{-1}$.

(b) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, prove that T satisfies a polynomial of degree n over F. **(10+10)**

22.(a) Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

(b) If $T \in A(V)$, prove that $T^* \in A(V)$. Also prove that for all $S, T \in A(V)$ and ª $\in F$,

(i) $(T^*)^* = T$

(ii) $(S+T)^* = S^* + T^*$

(iii) $(ªS)^* = \overline{ª}S^*$

(iv) $(ST)^* = T^*S^*$. **(10+10)**

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