

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2019

16/17/18UMT1MC01 – ALGEBRA AND CALCULUS - I

Date: 30-10-2019

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer **ALL** the questions.

(10 X 2 = 20)

1. Find the n th derivative of e^{ax} .
2. Find the subtangent and subnormal to the curve $y^2 = 4ax$ at any point.
3. Write the necessary conditions for the existence of a maxima or a minima of $f(x, y)$ at $x = a$ and $y = b$.
4. When is a point said to be a saddle point?
5. Write the formula for radius of curvature in polar coordinates.
6. Define evolute of the curve.
7. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \alpha^2$.
8. Define reciprocal equation.
9. State Descartes' rule of signs.
10. Find the number of positive roots in the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$.

PART - B

Answer any **FIVE** questions

(5 X 8 = 40)

11. If $y = x^2 e^x$, Show that $y_n = \frac{1}{2}n(n-1)y_2 - n(n-2)y_1 + \frac{1}{2}(n-1)(n-2)y$.
12. Find the slope of the tangent to the curve $r = a(1 - \cos \theta)$ at $\theta = \frac{\pi}{2}$.
13. Find the maximum and minimum value of $4x^2 + 6xy + 9y^2 - 8x - 24y + 4$.
14. Find the radius of curvature of the curve $xy^2 = a^3 - x^3$ at the point $(a, 0)$.
15. Find $p - r$ equation of the parabola $\frac{2a}{r} = 1 - \cos \theta$ with respect to the focus as pole.
16. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, given that $1 + \sqrt{-1}$ is a root of it.
17. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.
18. Solve $x^3 - 27x + 54 = 0$ by Cardon's method.

PART - C

Answer any **TWO** questions.

(2 X 20 = 40)

19. (a) Find the n^{th} differential coefficient of $x^2 \sin 5x$.

(b) Find the angle of intersection of curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.

(8+12)

20. (a) Using Lagrange's Multipliers, find the maximum value of $f(x, y, z) = x + y + z$

subject to $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

(b) Find the asymptotes of $x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0$.

(10+10)

21. Find the evolute of the parabola $y^2 = 4ax$ where $x = at^2$ and $y = 2at$

(20)

22. (a) If one of the roots of the equation $3x^5 - 4x^4 - 42x^3 + 56x^2 + 27x - 36 = 0$ is $\sqrt{2} + \sqrt{5}$,

find the other root.

(b) Find the positive root of the equation $x^3 + 6x - 2 = 0$ by Horner's method correct to two decimal places.

(8+12)
