

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



M.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – NOVEMBER 2019

18PMT3MC04 – ALGORITHMIC GRAPH THEORY

Date: 04-11-2019

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

ANSWER ALL QUESTIONS

I a) Define decomposition of a graph and give an example.

[OR]

b) Prove that a self complementary graph has either $4n$ or $4n + 1$ points. (5)

c) i) State and prove characterization theorem for trees.

ii) Prove that every closed odd walk contains an odd cycle. (10+ 5)

[OR]

d) (i) Define the structure of a hypercube and draw Q_4 .

(ii) Let G be a simple graph with vertices v_1, v_2, \dots, v_n and $n \geq 3$, prove that

$$e(G) = \frac{\sum e(G-v_i)}{n-2} \text{ and } d_G(v_j) = \frac{\sum e(G-v_i)}{n-2} - e(G-v_j).$$

(7+ 8)

II a) Show centre of a tree contains either one or two adjacent points.

[OR]

b) Find the number of different perfect matching's in $K_{n,n}$. (5)

c) i) State and prove Berge theorem.

ii) State and prove Halls theorem. (7+8)

[OR]

d) i) Show that closure of a graph is well defined

ii) State and prove Chavatal theorem for Hamiltonian graphs. (5+ 10)

III a) Write the applications of intersection graphs in traffic flow problems.

[OR]

b) Define interval graphs and give an example. (5)

c) i) State and prove characterization theorem for bipartite planar graphs.

ii) State and prove Euler's theorem for planar graphs. (7+8)

[OR]

d) i) Write BFS algorithms.

ii) State and prove characterization theorem for triangulated graphs. (5+ 10)

IV a) Define split graph and give an example.

[OR]

b) Give an example to show that the representation of split graph need not be unique. (5)

c) i) State and prove the characterization theorem for split graphs.

ii) Give an example of two non isomorphic split graphs having the same degree sequence.

(12 + 3)

[OR]

d) i) Define permutation labeling and give an example.

ii) Draw a permutation graph for the permutation $\pi = [4,3,6,1,5,2]$.

iii) Discuss about the application of permutation graphs in avoiding midair collisions of aeroplanes

(5+5+ 5)

V a) Give an example to show that circular-arc graph need not be an interval graph.

[OR]

b) Define unite interval graph and give an example. (5)

c) State and prove characterization theorem for circular-arc graphs. (10 + 5)

[OR]

d) State and prove Gilmore and Hoffman theorem for interval graphs. (15)
