LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2019**

PMT 1503 – ORDINARY DIFFERENTIAL EQUATIONS

Date: 01-11-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

ANSWER ALL QUESTIONS. EACH QUESTION CARRIES 20 MARKS.

1. (a) Let $x_p(t)$ be any particular solution of $L[x(t)] = d(t)$ and $x_h(t)$ be the general solution of $L[x(t)] = 0$. Show that $x(t) = x_p(t) + x_h(t)$ is the general solution of $L[x(t)] = d(t)$. (5)

(OR)

(b) By proving the necessary result, obtain the Abel's formula. (5)

(c) Using the method of variation of parameters, find the general solution of $x''' - x' = t$. (15)

(OR)

(d) Derive the various possible solutions of the equation $L(y) = a_0y'' + a_1y' + a_2y = 0$ where a_0, a_1, a_2 are known real constants and $a_0 \neq 0$. (15)

2. (a) Prove that (i) $P_l(-1) = (-1)^l$, and (ii) $P_l'(1) = \frac{1}{2}l(l+1)$. (5)

(OR)

(b) State and prove Rodrigues' formula. (5)

(c) Solve $2x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ by Frobenius method. (15)

(OR)

(d) State and prove the orthogonality properties of the Legendre polynomial. (15)

3. (a) When n is a non-zero integer, show that $J_{-n}(x) = (-1)^n J_n(x)$. (5)

(OR)

(b) Prove that $J_n'(x) = J_{n-1}(x) - \frac{n}{x}J_n(x)$. (5)

(c) State and prove the integral representations of Bessel function. (15)

(OR)

(d) Solve: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0, n \geq 0$. (15)

4. (a) Let x_m and x_n be two eigenfunctions of the Sturm- Liouville problem corresponding to two distinct eigenvalues λ_m and λ_n. Prove that that $[pW(x_m, x_n)]_A^B = 0$. (5)

(OR)

(b) Using the method of successive approximations, solve the initial value problem $x'(t) = -x(t), x(0) = 1, t \geq 0$. (5)

(c) State and prove Picard theorem for initial value problem. (15)

(OR)

(d) Prove that $x(t)$ is a solution of $L(x(t)) + f(t) = 0,\ a \le t \le b,$ iff $x(t) = \int_a^b G(t,s)f(s)\,ds$ where $G(t,s)$ is the Green function. (15)

5. (a) When do you say that a solution is stable? Check whether the solution of the equation $x' = -x$ is stable at origin. (5)

(OR)

(b) Define an autonomous system and state its stability behaviors. (5)

(c) Explain Lyapunov's direct method for analyzing the stability of $x' = Ax$. (15)

(OR)

(d) State and prove the two fundamental theorems on the stability behaviors of the non-autonomous systems. (15)

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