LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **PHYSICS**

FIRSTSEMESTER – APRIL 2017

16PPH1MC04/PH1820 - MATHEMATICAL PHYSICS - I

Date: 04-05-2017 Dept. No. Max. : 100 Marks
09:00-12:00

PART A

Answer all questions **10 x 2 = 20**

1. Verify whether $f(x) = (x^2)^{4+x^2}$ is solvable by numerical methods.
2. Sketch the graph $y = e^x$
3. Show that $Z = e^{i\propto}$ is an operator when operated on $f(z) = z_1$
4. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for $|z| = 3$.
5. Determine K for which the following vectors $\vec{u} = (3,2,1)$ and $\vec{v} = (-1,6,K)$ are orthogonal.
6. Let R^3 have the Euclidean inner-product. Find the angle between the vectors $\vec{u} = (-1,0,2)$ and $\vec{v} = (3,-5,7)$
7. Evaluate $\frac{\Gamma 6}{2\Gamma 3}$ using the knowledge of special functions.
8. Write the orthogonal relation for Bessel's polynomials.
9. i) Explain Cartesian tensors ii) Write Hooke's law in tensor notation.
10. Write the components of moment of inertia tensor.

PART B

Answer any four questions **4 x 7.5 = 30**

11. Using Newton-Raphson method, find the root of $(x^4 - x)^2 = 100$ which is near to $x = 2$ corrected up to 4 decimal places.
12. a) Derive Cauchy-Riemann equations.
    b) Check whether $f(z) = \frac{1}{z}$ is analytic or not in the region from $|z| < 5$
13. Show that the set of all solutions of the differential equation $p\frac{d^2y}{dx^2} + q\frac{dy}{dx} + ry = 0$ form a vector space.
14. a) Show that the contraction of the outer product of the two tensors A^l and B_m is an invariant quantity.
    b) Show that any inner product of the tensors A^l_m and B^{pq}_r is a tensor of a rank three.
15. Derive the orthogonality relation in Legendre polynomials.
16. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\sin x$, '$J$' stands for Bessel's function.

PART C

Answer any four questions **4 x 12.5 = 50**

17. Apply Euler's method to solve $\frac{dy}{dx} = x + 3y$ subject to $y(0) = 1$ and hence find the value of y when $x = 1$.

18. Evaluate $\int_{-\infty}^{+\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}$ using contour integration.

19. Find the eigen values and the corresponding eigen vectors for the matrix $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 4 & -2 & -4 \end{bmatrix}$

20. a) If A^{ij} is contra variant tensor, B_i is contra variant vector, then $A^{ij}\,B_k$ is a tensor of rank three and $A^{ij}\,B_j$ is a tensor of rank one.

    b) Find the metric tensor and the expression for the line element in cylindrical coordinates.

21. Show how Bessel's differential equation is obtained from Laplace's equation $\nabla^2 u = 0$ expressed in cylindrical coordinates (ρ, ϕ, z). Show that when, $\lambda\rho = x$, the above problem reduces to Bessel's equation.

22. Using contour integration, show that $\int_{-\infty}^{+\infty} \frac{x^2 dx}{x^4+1} = \frac{\pi}{\sqrt{2}}$

$$$$$$$$$$$$$$