## PH 4504 / PH 4502 / PH 6604 MATHEMATICAL PHYSICS

Date: 21-04-2017
09:00-12:00

Dept. No.

PART-A

## Answer ALL questions:

( $10 \times 2=20$ Marks $)$

1. Given $z_{1}=4-3 i$ and $z_{2}=2+5 i$, find imaginary part of $z_{1} z_{2}$.
2. Find the principal value of $\ln (1-i)$.
3. Integrate $\int_{-\pi i}^{\pi i} \cos z d z$
4. Evaluate $\int_{C} \frac{1}{z^{2}} d z$ where C is unit circle.
5. Determine the value of c if $u(x, t)=e^{-4 \pi t} \cos 4 x$ satisfies $\frac{\partial u}{\partial t}=c^{2} \frac{\partial^{2} u}{\partial^{2} x}$.
6. Write the two dimensional wave equation.
7. Define inverse Fourier transform of a function.
8. Give the change of scale property of a Fourier transform.
9. Define shift operator.
10. Write Simpson's $1 / 3$ rule.

## PART-B

## Answer any FOURquestions:

11. Find the real and imaginary parts of (a) $\sin z$ and (b) $\cosh z$.
12. Evaluate $\int_{C} \bar{z} d z$ from $z=0$ to $z=4+2 i$ along the curve C given by line from $z=0$ to $z=2 i$ and the line from $z=2 i$ to $z=4+2 i$.
13. Derive the wave equation for a vibrating string.
14. Find the Fourier cosine transform of $e^{-k x}, k>0$.
15. Compute the values of $y$ at $x=0.1,0.2,0.3$ for $y^{\prime}=x+y$ with $y(0)=0$ and $h=0.1$ using Euler's method.

PART-C

## Answer any FOURquestions:

(4 $\times 12.5=50$ Marks)
16. (a) Derive Cauchy-Riemann equations for a function $f(z)$ to be analytic.
(b) Show that $v=\cosh x \sin y$ is a harmonic function.
17. (a) State and prove Cauchy's integral theorem.
(b) Evaluate $\oint_{C} \frac{\sinh \pi z d z}{z^{2}-3 z}$ in counter clockwise where $\mathrm{C}:|\mathrm{z}-1|=1$.
18. Find the solution of two dimensional Laplace equation in electrostatic potential problem.
19. (a) State and prove convolution theorem for Fourier transforms.
(b) If $F(w)$ is the Fourier transform of $f(x)$, show thatF\{f $\left.f^{\prime}(\mathrm{x})\right\}=-\mathrm{w}^{2} \mathrm{~F}(\mathrm{w}) .(7.5+5)$
20. Derive Lagrange interpolation formula for unequal intervals and using it find $y(27)$ from the following table

| x | 14 | 17 | 31 | 35 |
| :--- | :--- | :--- | :--- | :--- |
| y | 68.7 | 64.0 | 44.0 | 39.1 |

