## LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **PHYSICS** 

THIRD SEMESTER - NOVEMBER 2016

PH 3506 – MATHEMATICAL PHYSICS

Date: 04-11-2016 Time: 09:00-12:00

## PART – A

## Answer ALL the questions:

- 1. Find the principal value of  $i^i$  where  $i = \sqrt{-1}$ .
- 2. State Cauchy's integral theorem.
- 3. Integrate  $xy \, dx \, dy$  between x = 0 to x = 1, y = 0 to y = 1.
- 4. Write the heat equation.
- 5. Determine the fundamental period of  $\cos \pi x$  and  $\sin x$ .
- 6. What is Fourier's integral?
- 7. Given  $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ , find  $A^T A$ , where  $A^T$  is the transpose of matrix A.

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- 8. If the det value of a  $2 \times 2$  matrix B is 4. What is the det value of 2B?
- 9. Write Newton's 'orward interpolation formula for equally spaced data.
- 10. Given y' = 1 + y with y(0) = 1 and step size as 0.2, find the value of y(0.2) using Euler's method.
  - PART B

## Answer any FOUR questions:

11. Determine the harmonic conjugate function of  $u = \sin x \cosh y$  and its corresponding analytic function.

12. State and prove Green's theorem in a plane.

13. Find the Fourier series of the function  $f(t) = \begin{cases} 0, & \text{if } -\frac{\pi}{\omega} < t < 0 \\ E \sin \omega t, & \text{if } 0 < t < \frac{\pi}{\omega} \end{cases}$ 

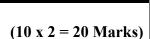
14. Show that  $N = \begin{pmatrix} \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta \end{pmatrix}$  satisfies Cayley-Hamilton theorem.

15. Find the Lagrange interpolating cubic polynomial for the following

х	0	3	5	6
y(x)	-6	6	104	196

and compute y(4).

16. Using Trapezoidal rule, evaluate  $\int_{0}^{1} \frac{dx}{1+x^2}$  by dividing the range into 10 equal parts.



 $(4 \times 7.5 = 30 \text{ Marks})$ 

Max.: 100 Marks

 $(4 \times 12.5 = 50 \text{ Marks})$ PART-C Answer any FOUR questions: 17. (i) Find the real and imaginary parts of tanh (x + iy). (ii) Integrate  $\oint \frac{(1+2z)\cos z}{(2z-1)^2} dz$  counterclockwise around the circle |z| = 1. (7.5+5)18. Given  $\vec{v} = y^2 \vec{i} + z^2 \vec{j} + x^2 \vec{k}$  and  $\phi = x + y + z$ , find the following (i) curl (grad  $\phi$ ) (ii)  $div (curl \vec{v})$  (iii)  $curl(\phi \vec{v})$ (4+4+4.5)19. Obtain the Fourier Cosine series and Fourier Sine series of the function  $f(x) = \begin{cases} \frac{2}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2}{L}(L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$ 20. Diagonalize the matrix  $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ . 21. Find the solution to four decimals of the following system of simultaneous equations by Gauss-Seidel iterative method. 10 x + 2 y + z = 92x + 20y - 2z = -44-2x + 3y + 10z = 2222. (i) State and prove Cauchy's integral formula. (ii) Find the directional derivative of f = xyz at the point (-1, 1, 3) in the direction of  $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}.$ (7.5+5)\*\*\*\*