LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **PHYSICS**

FIRSTSEMESTER – NOVEMBER 2017

17/16PPH1MC04 /PH1820- MATHEMATICAL PHYSICS - I

Date: 10-11-2017 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

PART A

Answer all questions **10 X 2 = 20**

1. Write the algorithm of Runge-kutta method of solving differential equations
2. Sketch the graph $y = \sin 3x$
3. If $f(z) = \ln(1+z)$, expand $f(z)$ in Taylor's series about $z = 0$.
4. Show that $\int f(z)dz$ is independent of the path followed Cauchy's theorem
5. Show that there is no scalar $\alpha_1, \alpha_2, \alpha_3$ such that $\alpha_1(1,0,1,0) + \alpha_2(1,0,-2,1) + \alpha_3(2,0,1,2) = (1,-2,2,3)$
6. Evaluate $||\vec{u} + \vec{v}||$ for $\vec{u} = (1,0,-1,2) and\ \vec{v} = (2,1,3,-1)$
7. If x^i and x^{-i} are independent coordinates of a point, $\frac{\partial x^i}{\partial x^{-P}} . \frac{\partial x^{-P}}{\partial x^j} = \delta^i_j$
8. Define piezoelectric tensor
9. Find $\Gamma(\frac{-1}{2})$
10. Arrive at the solution of $\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta$ using the knowledge of special functions

PART B

Answer any four questions 4 X 7.5 = 30

11. Find the real root of the equation, $x^3 - 2x - 5 = 0$, correct up to three decimal places using Newton-Raphson method.
12. Evaluate $\oint \frac{dz}{z-2}$ around a i) circle $|z-2| = 4$ and ii) $|z| = 25$
13. Show that a set $S = \{(1,2,1), (2,1,0), (1,-1,2)\}$ forms a basis in IR^3
14. Define the term contraction. Get the inner product and outer product of tensors $A^l_m\ and\ B^{pq}_r$
15. Prove that $\frac{1}{\sqrt{1-2xt+\ ^2}} = \sum_{n=0}^{\infty} P_n t^n$, where P_n stands for Legendre polynomials
16. Evaluate $J_{1/2}(x)$, where J stand for Bessel's functions.

PART C

Answer any four questions **4 X 12.5 = 50**

17. Find the Eigen values and Eigen vectors of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

18. Evaluate, using contour integration $\int_0^\infty \frac{dx}{x^6+1}$

19. Verify that the set of vectors $\{\vec{u}_1 = (1,0,0,-1), \vec{u}_2 = (0,1,0,-1)\ and \vec{u}_3 = (0,0,1,-1)\}$ is a basis of the solution space of the equation $x_1 + x_2 + x_3 + x_4 = 0$ which is a subspace of IR^4 and orthogonalize this basis by using Gram Schmidt process. (Use Euclidean inner product).

20. Obtain an expression for line element in Euclidian space. Determine the conjugate metric tensor in spherical coordinates

21. Prove that $e^{\frac{x}{2}(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} J_n(x)t^n$

22. i) Prove that $u = e^{-x}(x\sin y - y\cos y)$ is harmonic.

    ii) Evaluate $\oint \frac{sin^6 z}{(z-\frac{\pi}{6})^3} dz$ if c is the circle $|z| = 7$

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