# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600 034 **B.Sc.**DEGREE EXAMINATION –**PHYSICS** THIRD SEMESTER – NOVEMBER 2017 PH 3506– MATHEMATICAL PHYSICS Dept. No. Date: 04-11-2017 Max.: 100 Marks Time: 09:00-12:00 PART – A Answer ALL the questions: (10 x 2 = 20 Marks)1. Is $f(z) = \sin z$ an analytic function? 2. Express $f(z) = \frac{1}{3+4i}$ in the form of a+ib 3. Define symmetric and antisymmetric matrices. Prove that the eigenvalues of a Hermitian matrix are real. 4. 5. Check whether $\vec{A} = \vec{12i} + \vec{4j} - \vec{6k}$ and $\vec{B} = \vec{6i} + \vec{2j} - \vec{3k}$ are parallel or perpendicular. 6. Define a periodic function with an example. 7. Write down Simpson's one third rule.

- 8. Define the gradient of a scalar function.
- 9. Define odd and even functions and give one example each.
- 10. Write the algorithm of Euler's method.

## PART – B

## Answer any four questions:

11. Solve the equations using matrices

4x+2y+z+3u=0; 6x+3y+4z+7u=0; 2x+y+u=0.

12. a) State and prove Cauchy's Integral Theorem.

b) Using Cauchy's Integral Formula, evaluate 
$$\iint_{c} \frac{2z^2 + z}{z - 1} dz; [z] = 7.$$
 (2.5)

13. If  $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$  Show that (i) curl  $\vec{r} = 0$  (ii) curl  $(\vec{r} / r^3) = 0$ . (4+3.5)

14. Find the solution of dy/dx = x+y from x=0 to 0.2 by Euler's method.

- 15. Prove that div curl A = 0; curl grad  $\varphi = 0$ . (4+3.5)
- 16. Obtain a Fourier series for  $f(x) = e^x \pi < x < \pi$ .

 $(4 \times 7.5 = 30 \text{ Marks})$ 

(5)

#### PART – C

#### Answer any four questions:

1

17 Find the eigenvalues and the normalized eigenvectors of A =  $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ .

18. Derive the trapezoidal rule. Use trapezoidal and simpson's one third rule to evaluate the approximate

value of 
$$\int_{0} dx(1+x)$$
 corrected to 3 decimal places taking h=0.25 (4.5+4+4)

19. Derive the Cauchy - Riemann equations in polar form.

(6.5 + 6)

20. State and prove Gauss Divergence theorem. Using it evaluate

 $\iint (x^3 \, dy \, dz + y^3 \, dz \, dx + z^3 \, dy \, dx)$  Over the surface of the sphere S of radius. (6.5+6)

21. Find the Fourier series expansion of the periodic function having period  $2\pi$  where

 $f(x) = x^2 - \pi \le x \le \pi$ . Hence find the sum of the series  $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$ 

22 Verify Cayley Hamilton theorem for the matrix  $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{pmatrix}$ .

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(4 x 12.5 = 50 Marks)