LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

SECOND SEMESTER – **APRIL 2015**

ST 2816 - SAMPLING THEORY

Date : 21/04/2015 Dept. No. Max. : 100 Marks

Time : 01:00-04:00

SECTION – A

Answer **ALL** questions. Each carries TWO marks. **(10 x 2 = 20 marks)**

1. Explain Probability Sampling Design. What is the meaning of Probability Sampling?
2. Define Mean Square Error of an estimator. Express it in terms of Variance and Bias of the estimator.
3. Find variance and covariance of inclusion indicators.
4. Define first and second order inclusion probabilities and write their formula under SRSWOR.
5. Describe Cumulative Total Method and verify whether or not this method is a PPS selection method.
6. Why do we discard Hurwitz – Thompson estimator when using PPSWOR sampling scheme?
7. Describe Linear Systematic Sampling Scheme and give its sampling design.
8. Given the Sample size n = 8 and the population size N = 40. Write all possible samples under Modified Systematic Sampling Scheme.
9. Under SRSWOR scheme, find the approximate Bias and MSE of $\hat{Y}_R$.
10. Explain Ratio Estimation under Double Sampling.

SECTION – B

Answer any **FIVE** questions. Each carries EIGHT marks. **(5 x 8 = 40 marks)**

11. Under a given sampling design, show that one can find more than one unbiased estimator for a given parameter.
12. Derive estimated variance $v(\hat{Y}_{HT})$ under any design P(·).
13. Under SRSWOR, find $Cov_p\left(\frac{\hat{}}{Y}, \frac{\hat{}}{X}\right)$, where $\frac{\hat{}}{Y} = \frac{1}{n}\sum_{i\in s} Yi$ and $\frac{\hat{}}{X} = \frac{1}{n}\sum_{i\in s} Xi$.

    are the sample means and (X_i, Y_i), $i \in s$ is the pair of values associated with the i^{th} sampled unit with respect to the variables x and y.
14. Explain Lahiri's method of sampling and verify whether or not this method is a PPS selection method.
15. Under PPSWOR sampling scheme, define Des Raj ordered estimator for population total and find its mean.
16. In Centered Systematic Sampling, when the population is linear, check whether Or not the usual expansion estimator is unbiased for the population total.
17. Estimate the proportion Π_A of the persons having a sensitive characteristic A in a given population, using Warner's Randomized Response Technique.
18. For estimating population total, obtain Hartley – Ross unbiased ratio type estimator.

SECTION – C

Answer any **TWO** questions. Each carries **TWENTY** marks. **(2 x 20 = 40 marks)**

19 (a) After the decision to take a SRS has been made, it was realized that Y_1, the value of unit with label 1 would be unusually low and Y_N, the value of unit with label N would be unusually high. In such cases it is decided to use the estimator

$$\frac{\hat{}}{Y}^* = \begin{cases} \frac{\hat{}}{Y} + c, & \text{if } 1 \in s, \quad N \notin s \\ \frac{\hat{}}{Y} - c, & \text{if } 1 \notin s, \quad N \in s \\ \frac{\hat{}}{Y}, & \text{otherwise,} \end{cases}$$

where 'c' is a pre-determined constant. Show that (i) $\frac{\hat{}}{Y}^*$ is unbiased for $\overline{Y}$ for any 'c'. (ii) Derive V ($\frac{\hat{}}{Y}^*$). (iii) Find the value of c for which $\frac{\hat{}}{Y}^*$ is more efficient than $\frac{\hat{}}{Y}$. **(14)**

19 (b) Under Midzuno Sampling Design, obtain the first and second order inclusion probabilities. **(6)**

20 (a) For n = 2, find the population variance V($\hat{Y}_{DR}$). **(12)**

20 (b) Discuss about the utility of auxiliary information and describe any one estimation method in which the auxiliary information is used. **(8)**

21. Discuss about Two Phase Sampling. Suppose samples are drawn using SRS in both the phases of sampling, suggest $\hat{X}$, $\hat{Y}$ and $\hat{X}_d$ when
(i) the second phase sample is a sub-sample of the first phase sample,
(ii) the second phase sample is independent of the first phase sample.

Under the above two cases, obtain V($\hat{X}$), V($\hat{Y}$), V($\hat{X}_d$), Cov($\hat{X}$, $\hat{Y}$), Cov($\hat{X}$, $\hat{X}_d$), and Cov($\hat{Y}$, $\hat{X}_d$). **(20)**

22 (a) Under PPSWR sampling scheme, find the mean E ($\hat{Y}_{HHE}$), population variance V($\hat{Y}_{HHE}$) and estimated variance v($\hat{Y}_{HHE}$). **(12)**

22 (b) In Stratified Random Sampling, write a note on proportional allocation for a given cost. Hence find V($\hat{Y}_{St}$) under proportional allocation. **(8)**

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