# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **STATISTICS**

**FIRST** SEMESTER – **APRIL 2016**

**ST 1822 - STATISTICAL MATHEMATICS**

Date: 02-05-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer **all** the questions.

1. a) For any $a, b \in R$, prove that $\left||a| - |b| \le |a-b|\right|$. Also prove that $\{|s_n|\}$ converges to $|L|$ if $\lim_{n\to\infty} s_n = L$.

**OR**

b) If $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then prove that $\lim_{n\to\infty}(s_n + t_n) = (L+M)$.

**(5)**

c) (i) Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ converges.

(ii) If $\sum a_n$ is dominated by $\sum b_n$ and if $\sum b_n$ converges absolutely, then prove that $\sum a_n$ converges absolutely. **(10+5)**

**OR**

d) State and prove Leibnitz theorem. **(15)**

2) a) Verify the hypothesis and the conclusion of the mean value theorem for the following functions $f(x) = logx$ in $[1, e]$ and $f(x) = \frac{x}{x-1}$ in $2 < x < 4$.

**OR**

b) If the real valued function $f$ is differentiable at the point $a \in R$ then prove that $f$ is continuous at '$a$'.

**(5)**

c) (i) State and prove Taylor's theorem.

(ii) Define continuity, jump discontinuity and removable discontinuity. **(10+5)**

**OR**

d) (i) State and prove Mean value theorem for derivatives.

(ii) State and prove inverse function theorem. **(7+8)**

3. a) For any partition P of $[a, b]$, prove that $m[f; P](b-a) \le L[f; P] \le U[f; P] \le M[f; P](b-a)$.

**OR**

b) If $f \in R[a, b]$ is continuous at $x_0 \in [a, b]$ and if $F(x) = \int_a^x f(t)dt$ where $a \le x \le b$ then prove that $F'(x_0) = f(x_0)$. **(5)**

c) (i) Let $f$ be bounded function on the closed bounded interval $[a, b]$ then prove that $f$ is Riemann integrable if and only if for every $\varepsilon > 0$ there exists a subdivisions $P$ of $[a, b]$ such that $U[f; P] - L[f; P] < \varepsilon$.

(ii) Test the convergence of the following integrals (a) $\int_1^{\infty} \frac{1}{x^2} dx$ and (b) $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$ **(8+7)**

**OR**

d) (i) State and prove First Fundamental theorem of Calculus.

(ii) If $f'$ and $g'$ are continuous on $[a, b]$, then prove that $\int_a^b f(x) g'(x) dx = f(b)g(b) - f(a)g(a) - \int_a^b f'(x)g(x)dx$. **(10+5)**

4. a) Prove that a square matrix A is singular if and only if its columns are linearly dependent.

**OR**

b) State and prove Cauchy-Schwarz inequality. **(5)**

c) (i) ) If the linear system of $m$ equations in $n$ unknowns $AX + B = 0$ is consistent then prove that a complete solution thereof is given by a complete solution of the corresponding homogeneous system $AX = 0$ plus any particular solution of $AX + B = 0$.

(ii) If the $k$ $n$-vectors $A_1, A_2, \dots A_k$ are linearly independent but the vectors $A_1, A_2, \dots A_k, B$ are linearly dependent then prove that $B$ is a linear combination of $A_1, A_2, \dots A_k$.

**(8+7)**

OR

d) (i) Let $V$ be a vector space over $F$, not consisting of the zero vector alone then prove that $V$ contains atleast one set of linearly independent vectors $A_1, A_2, \dots, A_k$ such that the collection of all linear combinations $X$ of the form $X = t_1A_1 + t_2A_2 + \cdots + t_kA_k$ where $t's$ are arbitrary scalars from $F$, is precisely $V$. Moreover, prove that the integer $k$ is uniquely determined for each $V$.

(i) Express the vector $(1, -2, 5)$ as a linear combination of the vectors $(1,1,1)$, $(1,2,3)$ and $(2, -1,1)$ in $R^3$, where $R$ is the field of real numbers. **(10+5)**

5 a) Prove that the characteristic vectors associated with distinct characteristic roots of a real symmetric matrix A are orthogonal.

**OR**

b) Let $A$, $P$ be an $n \times n$ matrix. Then prove that $A$ and $P^{-1}AP$ have the same characteristic equation

**(5)**

c) State and prove Gram Schmidt orthonormalization process theorem and hence apply it to the vectors $(1,0,1), (1,0,-1), (0,3,4)$ to obtain an orthonormal basis for $R^3$. **(15)**

OR

d) Reduce the matrix $A = \begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$ to diagonal form. **(15)**

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