LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2016**

ST 2815 - TESTING STATISTICAL HYPOTHESIS

Date: 20-04-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION – A : ANSWER ALL THE QUESTIONS (10 X 2 = 20)

1 Write any two examples for simple and composite hypothesis.
2 Define uniformly most powerful test.
3 State Generalized Neyman-Pearson Theorem
4 What is MLR property?
5 Define one parameter exponential family.
6 Define α-similar test
7 Justify the following statement: “A test with Neyman structure is α similar”
8 Give an example of an invariant decision problem
9 When do we say function is maximal invariant?
10 Briefly explain the principles of LRT

SECTION – B: ANSWER ANY FIVE THE QUESTIONS (5X 8 = 40)

11 if $X \sim B(1,\theta)$, $\theta = 0.25, 0.5$ and for testing $H: \theta = 0.5$ against $K: \theta = 0.25$.
Let $\phi_1(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x = 1 \end{cases}$ and $\phi_2(x) = \begin{cases} 0.1 & \text{if } x = 0 \\ 0.0 & \text{if } x = 1 \end{cases}$ **(8)**
Calculate Size and Power of the test function

12 Let X be random variable with probability mass function under H and K are given by

X	1	2	3	4	5	6
$f_0(x)$	0.01	0.01	0.01	0.01	0.01	0.95
$f_1(x)$	0.05	0.04	0.03	0.02	0.01	0.85

(8)
Suppose α = 0.03, find the test function by using Nayman-Pearson’s lemma and find the probability of Type II error and Power of the test.

13 Let β denote the power of a most powerful test of level α for testing simple hypothesis H against simple alternative K. Prove that (i) β ≥ α and (ii) α < β unless $p_0 = p_1$. **(8)**

14 Let $X_1, X_2, \ldots, X_n$ be a random sample from a Cauchy distribution with parameter (1, θ). Show that this family does not have MLR property. **(8)**

15 Derive UMPU level α test for testing the hypothesis H: $\theta_1 \le \theta \le \theta_2$ against K: $\theta < \theta_1$ or $\theta > \theta_2$ for one parameter exponential family. **(8)**

16 Let X have the distribution $P \in \mathcal{P}$ and T be a sufficient statistic for $\mathcal{P}$. Show that a necessary and sufficient condition for all similar tests have Neyman structure is that the family $\mathcal{P}^T$ of distributions of T is boundedly complete **(8)**

17 Let $X_1, X_2, \ldots, X_m$ be a random sample from P(λ) and $Y_1, Y_2, \ldots, Y_n$ be a random sample from P(μ). Derive UMPU level α test for testing the hypothesis H: λ ≤ μ against K: λ > μ. **(8)**

18 Obtain the Likelihood ratio Test for equality of means of ‘k’ Normal populations with a common variance. **(8)**

SECTION – C: ANSWER ANY TWO QUESTIONS **(2X 20= 40)**

19 State and prove the necessary and sufficient condition of Nyman – Pearson fundamental Lemma. **(20)**

20 a) Show that a necessary and sufficient condition for the family of distribution to have MLR property is that $\frac{\partial \log f(x,\theta)}{\partial\theta\partial x}$ exists and is non-negative. **(10)**

b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from $f(x,\theta) = e^{-(x-\theta)}; \theta < x < \infty$. **(10)**

Does $\{f_\theta(x)\}$ belong to the exponential family? Does $\{f_\theta(x)\}$ have MLR?

21 a) If $X_1, X_2, \ldots, X_n$ be a random sample from N(μ, σ^2), with both parameters are unknown. Derive Likelihood Ratio Test of level α for testing H: $\sigma^2 = \sigma_0^2$ versus K: $\sigma^2 \neq \sigma_0^2$ **(10)**

b) Define multi Parameter exponential family. Also mention its objectives and properties. **(10)**

22 If X ~ B(m, p_1) and Y ~ B(n, p_2) are independent, where m ≠ n, then Derive the UMPUT of size – α for testing H: $p_1 \leq p_2$ against K: $p_1 > p_2$. **(20)**