LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER - APRIL 2016

ST 2962 – MODERN PROBABILITY THEORY

Date: 27-04-2016 Time: 01:00-04:00

SECTION A

Answer ALL of the following.

- 1. Can the union of two fields be a field? Explain.
- 2. Define: Monotone Field.
- 3. Define: Discrete Probability Space.
- 4. Give an example for the minimal field containing the class itself.

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- 5. Define Mixture of Distributions.
- 6. Let X be a continuous Gamma Variate, derive, $M_x(\theta)$.
- 7. Derive the Harmonic Mean of Beta Distribution of first kind.
- 8. Explain almost sure convergence.
- 9. When will you say, a random variable is centered at some constant c and its expectation?
- 10. Explain law of large numbers?

SECTION B

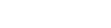
Answer any FIVE from the following

- 11. Let ξ_i be the class of all intervals of the form (a, b), (a<b) a, b ξ R, but arbitrary. Then P.T. $\sigma(\xi_i) = B$.
- 12. Explain: Induced Probability Space with an example.
- 13. Show that Standard Gamma variate converges in distribution to Standard Normal variate.
- 14. If $\sum \sigma_n^2 < \infty$ then, prove that, $\sum (X_k E(X_k))$ converges in probability.
- 15. State and prove the properties of Expectation of Non negative random variables.
- 16. If $X_n \xrightarrow{p} X$ then, show that there exists a subsequence $\{X_{nk}\}$ of $\{X_n\}$ which converges a.s. to X.
- 17. State and prove the properties of characteristic function.
- 18. Explain in detail the applications of Central limit theorem.

SECTION C

Answer any TWO from the following

- 19. i) Prove that The intersection of arbitrary number of fields is a field.
 - ii) Prove that $X_n \xrightarrow{p} c$ implies that $F(X_n) \longrightarrow 0$ for x < c, $F(X_n) \longrightarrow 1$ for x < c and conversely. (10)



Max.: 100 Marks

(10x2=20 marks)



(2x20=40 marks)

(10)

(5x8=40 marks)

| | 20. i) Let $X_n \xrightarrow{L} X, Y_n \xrightarrow{L} c$, then P.T. a. $X_n + Y_n \xrightarrow{L} X + c$ b. $X_n Y_n \xrightarrow{L} cX$ | |
|-----|--|--------------|
| | c. $X_n / Y_n \xrightarrow{L} X / c$. | (10) |
| | ii) Let $\{x_n\}$ be a sequence of i.i.d. r.v.'s with characteristic function $\varphi(u)$. Then $S_n/n \xrightarrow{p} E(u)$ | (X). (10) |
| 21. | i) Derive the Kolmogorov Inequalities. | (10) |
| | ii) State and prove the necessary and sufficient condition for a series of random variables to converge a.s. | (10) |
| 22. | i) State and prove Liapounov's theorem. | (12) |
| | ii) Explain Central limit theorem as a generalisation of law of large numbers. | (8) |

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