LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **APRIL 2016**

ST 3815 - MULTIVARIATE ANALYSIS

Date: 25-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

SECTION – A

Answer ALL the questions **(10 x 2 = 20)**

1. Let X,Y and Z have trivariate normal distribution with null mean vector and Covariance matrix $\begin{bmatrix} 2 & 5 & 0 \\ 5 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$, find the distribution of X+Y.
2. Mention any two properties of multivariate normal distribution.
3. Write down the characteristic function of a multivariate normal distribution.
4. Explain the use of the partial and multiple correlation coefficients.
5. Comment on repeated measurements design.
6. Describe a) Common factor and b) Communality.
7. Explain the classification problem into two classes.
8. Find the maximum likelihood estimates of the 2×1 mean vector μ and 2×2 covariance matrix Σ based on random sample $X' = \begin{pmatrix} 6 & 8 & 10 & 8 \\ 12 & 8 & 14 & 14 \end{pmatrix}$ from a bivariate population.
9. Outline the use of Discriminant analysis.
10. Write a short note on data mining.

PART– B

Answer anyFIVE questions **(5X8=40 marks)**

11. Find the multiple correlation coefficient between X_1 and $X_2, X_3, \ldots, X_p$. Prove that the conditional variance of X_1 given the rest of the variables cannot be greater than unconditional variance of X_1.
12. Derive the characteristic function of multivariate normal distribution.
13. Let $X_1, X_2, ..., X_n$ be independent $N(0, \sigma^2)$ random variables. Show that $(X' AX)/\sigma^2$ is chi-square if A is idempotent where $X = (X_1, X_2, ..., X_n)'$.
14. Obtain the linear function to allocate an object to one of the two given normal populations.
15. Let $X \sim N_p(\mu, \Sigma)$. If $X^{(1)}$ and $X^{(2)}$ are two subvectors of X, obtain the conditional distribution of $X^{(1)}$ given $X^{(2)}$.
16. Giving suitable examples explain how factor scores are used in data analysis.

17. Let $(X_i, Y_i)'$ i = 1,2,3 be independently distributed each according to bivariate normal with mean vector and covariance matrix as given below. Find the joint distribution of the six variables. Also find the joint distribution of $\overline{X}$ *and* $\overline{Y}$.

Mean Vector: $(\mu, \tau)'$, covariance matrix: $\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$.

18. Prove that the extraction of principal components from a dispersion matrix is the study of characteristic roots and vectors of the same matrix.

PART– C

Answer anyTWO questions **(2 X 20 =40marks)**

19. Derive the distribution function of the generalized T^2 – Statistic.

20. a) Define generalized variance.

b) Show that the sample generalized variance is zero if and only if the rows of thematrix of deviation are linearly dependent.

c) Find the covariance matrix of the multinormal distribution which has the quadratic form $2{x_1}^2 + {x_2}^2 + 4{x_3}^2 - x_1x_2 - 2x_1x_3$. **(3+12+5)**

21. a) Outline single linkage and complete linkage clustering procedures with an example.

b) If $X \sim N_p(\mu, \Sigma)$ then prove that $Z = DX \sim N_p(D\mu, D\Sigma D')$ where D is qxp matrix rank q≤p. **(10+10)**

22. a) Let $Y \sim N_p(0, \Sigma)$. Show that $Y'\Sigma^{-1}Y$ has χ^2 distribution.

b) Prove that under some assumptions (to be stated), variance covariance matrix can be written as $\Sigma = LL' + \Psi$ in the factor analysis model. Also discuss the effect of an orthogonal transformation. **(10+10)**

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