LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **APRIL 2016**

ST 3816 – STOCHASTIC PROCESS

Date: 27-04-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART A

Answer ALL the questions: **(10 x 2 =20 marks)**

1) Define transient state and recurrent state of a Markov Chain.
2) Obtain the periodicity of a Markov chain with status 0,1 and transition probability

$$P = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 0 \end{bmatrix}$$

3) Define a renewal process.
4) Define a martingale $\{X_n\}$ with respect to $\{Y_n\}$.
5) Write the PGF of a Poisson process.
6) Show that communication of a Markov chain satisfies transitivity.
7) Write the density of the inter arrival time in a Poisson process.
8) Explain branching processes.
9) Define excess life and current life.
10) Define a stationary process.

PART B

Answer any FIVE questions: **(5 x 8 =40 marks)**

11) Explain discrete queueing Markov chain.
12) State and prove Chapman-Kolmogrov equation.
13) Derive the differential equations for a pure birth process.
14) Show that $X_n = (\sum Y_k)^2 - n\sigma^2$ is a martingale if Y_i are iid $E(Y_i) = 0$, $Y_0 = 0$ and $E(Y_i^2) = \sigma^2$.
15) Compute the limiting distribution π_i, i = 0, 1, 2 for the transition probability matrix,

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \end{bmatrix}$$

16) For a renewal process show that $M(t) = F(t) + F*M(t)$.
17) For a renewal process, obtain $E[W_{N(t)+1}]$.
18) If $i \leftrightarrow j$ then show that $d(i) = d(j)$.

PART C

Answer any TWO questions: **(2 x 20 =40 marks)**

19) a) Show that a state i is recurrent if and only if $\sum_n P_{ii}^n = \infty$.

b) Show that one dimensional random walk on the set of integers is a Markov chain.

20) a) Let P be a regular transition probability matrix on the status 0, 1, 2, then show that the limiting distribution satisfies $\pi_i = \sum_k \pi_k p_{ki}$ and $\sum_k \pi_k = 1$.

b) Show that for $P = \begin{bmatrix} 1-a & b \\ b & 1-b \end{bmatrix}$ the n step transition probability is

$$P^n = \frac{1}{a+b}\begin{bmatrix} b & a \\ b & a \end{bmatrix} + \frac{(1-a-b)^n}{a+b}\begin{bmatrix} a & -a \\ -b & b \end{bmatrix}.$$

21) a) State the postulates of a Poisson process. Derive $P_n(t)$ for a Poisson process.

b) Let $X_1(t)$ and $X_2(t)$ be two independent Poisson processes. Obtain the conditional distribution of $X_1(t)$ given $X_1(t) + X_2(t) = n$.

22) a) Obtain the probability generating function relations for a Branching process.

b) Discuss about the probability of extinction in a Branching process.

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