LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHS, PHYSICS, PLANT BIO. & ADV. ZOO**

FOURTH SEMESTER – **APRIL 2016**

ST 4209 / ST 4206 - MATHEMATICAL STATISTICS

Date: 27-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Section – A

Answer all the questions **10 X 2 = 20 marks**

1. Define probability of an event.
2. Write the sample space for throwing two fair dice.
3. Define the distribution function of a random variable.
4. If X has the p.d.f. $f(x) = c\,x\,e^{-x}$, $0<x<\infty$, zero elsewhere , find c.
5. If P(A) =1/2 and P(B) = 1/3 and $P(A\cap B) = 1/5$, compute $P(A^c \mid B^c)$.
6. Define bivariate normal distribution.
7. State Bayes' theorem.
8. Define marginal and conditional distributions.
9. When an estimator is said to be good?
10. Define probability of type I and type II errors.

Section – B

Answer any five questions **5 X 8 = 40 marks**

11. Let $p(x) = (1/2)^x$, x = 1,2,3,… , zero elsewhere , be the probability mass function of the random variable X. Find the moment generating function , the mean and the variance of X.
12. State and prove Chebyshev ' s inequality.
13. Show that under certain conditions binomial distribution tends to Poisson.
14. Derive the M.G.F. of exponential distribution and hence find mean and variance.
15. Let $Y_1< Y_2<Y_3<Y_4$ denote the order statistics of a random sample of size 4 from a distribution having p.d.f. f(x) = 2x , $0<x<1$, zero elsewhere. Find $P(Y_3 > 1/2)$.
16. Let X have the p.d.f. f(x) = 1 , $0<x<1$, zero elsewhere . Show that Y = -2 ln X has a chi-square distribution with 2 degrees of freedom.
17. Show that the random variables X_1 and X_2 with joint p.d.f. $f(x_1 , x_2) = 12x_1x_2(1-x_2)$, $0 <x_1<1$, $0<x_2<1$, zero elsewhere, are stochastically independent.
18. If $X_1, X_2,…,X_n$ is a random sample from normal distribution with mean θ and variance 1 , find the maximum likelihood estimator of θ.

Section-C

Answer any two questions **2 X 20 = 40 marks**

19. (a) State and prove addition theorem on probability for n events.(10 marks)
    (b) State and prove Boole's inequality.(10 marks)
20. (a) Show that for normal distribution, quartile deviation is (2/3) σ. (8 marks)
    (b) If X is normal with mean 75 and variance 100, find (i) P(X < 60) (ii) P(70< X <100) (iii) P(X > 65). (12 marks)
21. Let $f(x_1,x_2) = 21x_1^2x_2^3$, $0<x_1<x_2<1$, zero elsewhere be the joint p.d.f. of X_1 and X_2. Find the conditional mean and variance of X_1 given $X_2=x_2$, $0<x_2<1$ and X_2 given $X_1=x_1$, $0<x_1<1$. **(20 marks)**
22. (a) Derive the probability density function of Student's t distribution. **(15 marks)**
    (b) Explain the methods of maximum likelihood and moments. **(5 marks)**

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