LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – **APRIL 2016**

ST 5400 - APPLIED STOCHASTIC PROCESSES

Date: 29-04-2016 Dept. No. ☐ Max. : 100 Marks
Time: 01:00-04:00

PART A

Answer ALL the questions: (10 X 2 =20)

1) Define a Markov Chain.
2) Define periodicity of a state in a Markov chain.
3) Which one of the state is recurrent in the Markov chain

$$P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 1 & 0 \\ 1 & 1/2 & 1/2 \end{array}$$

4) Define a transient state of a Markov chain.
5) Define a process with independent increments.
6) Define n-step transition probability matrix.
7) Define a stationary distribution.
8) Define mean recurrence time of a Markov chain.
9) State the mean and variance of a Poisson process.
10) Write the distribution of inter arrival time in a Poisson process.

PART B

Answer any FIVE questions: (5 X 8 =40)

11) Show that Markov chain is completely determined if the initial distribution and the transition probability matrix is given.
12) Given the Markov chain with $P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1/2 & 1/2 \\ 1 & 1/2 & 0 & 1/2 \\ 2 & 1/2 & 1/2 & 0 \end{array}$, $p[X_0 = i] = 1/3, i = 1, 2, 3$

    Obtain (i) $P[X_3=0, X_2=1, X_1 = 2, X_0 = 0]$, (ii) $P[X_2=1]$ and (iii) $P[X_2=1| X_1=0, X_0=2]$.

    (2+4+2)
13) State and prove Chapman-Kolmogrov equation.
14) Given $i\leftrightarrow j$, show that when i is recurrent then j is also recurrent.
15) State and prove the additive property of Poisson process.
16) Obtain the differential equations for a pure birth process.
17) Show that communication is an equivalence relation.
18) Show that in a finite irreducible Markov chain all the states are recurrent.

PART C

Answer any TWO questions: (2 X 20 =40)

19) a) Define a Stochastic process, Index set, State space and obtain the classifications.

b) Explain how the inventory model can be viewed as a Markov chain.

20) State the postulates of a Poisson process and derive the Poisson process.

21) a) A housewife buys 3 kinds of cereals A, B and C. She never buys the same cereals in successive weeks. If she buys cereals A in a week, the next week she buys B. However if she buys B or C, the next week she is 3 times as likely to buy A as the other cereal. Obtain the transition probability matrix. Find which of the states are recurrent and transient and also find the periodicity of all the states and obtain the stationary distribution.

22) a) State Basic limit theorem.

b) Consider the Markov chain with states 0, 1, 2, 3 and the transition probability matrix

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

Verify all the conditions of Basic limit theorem. Obtain the stationary distribution.

(5+15)

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