LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECONDSEMESTER – APRIL 2017

16PST2ES02- MODERN PROBABILITY THEORY

Date: 28-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

Section A

Answer ALL questions(10 X 2 = 20)

1. Define probability measure.
2. What do you mean by an Induced probability space?
3. Define Lebesgue-Stieltjes measure.
4. Define distribution function of a random variable.
5. Define decomposition of distribution functions.
6. Define independence of n random variables.
7. What are the three types of random variables?
8. Define bivariate characteristic function.
9. Define convergence in distribution.
10. Define stability of independent random variables.

Section B

Answer ANY FIVE questions. **(5 X 8 = 40)**

11. Define a Borel sigma field and show that any interval is a Borel set, but the converse is not true.

12. Let F be the distribution function on R given by,

$$F(x)=\begin{cases} o, & if\ x<-1 \\ 1+x, & if\ -1\le x<0 \\ 2+x^2, & if\ 0\le x<2 \\ 9, & if\ x\ge 2 \end{cases}$$

If μ is a Lebesgue-Stieltjes measure corresponding to F, compute the measure of each of the following sets. **(i)** {2}, **(ii)** [-1/2, 3), **(iii)** (-1, 0]U(1, 2),
(iv) [0,1/2)U(1,2).

13. State and prove the continuity property of probability.

14. State and prove **(i)** Minkowski Inequality and **(ii)** Jensen's Inequality

15. State the inversion theorem for discrete and continuous case and find the distribution if $\varphi(u) = e^{-|t|}$, $-\infty < t < \infty$.

16. Define convergence in probability and state and prove the criterion for convergence in probability

17. State and prove weak law of large numbers for the non iid case.

18. State and prove Markov's theorem.

Section C

Answer ANY TWO questions. **(2 X 20 = 40)**

19. **(i)** Define independence of events, independence of classes, independence of random variables.

(ii) State and prove the two necessary and sufficient conditions for n random variables to be independent.

(6 + 14)

20. **(i)** Show that convergence in probability implies convergence in distribution.

(ii) Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$. Then prove that

(a) $a X_n \xrightarrow{P} a X$

(b) $X_n + Y_n \xrightarrow{P} X + Y$

I $X_n \ Y_n \xrightarrow{P} X \ Y$

(d) $\frac{X_n}{Y_n} \xrightarrow{P} \frac{X}{Y}$, P[Y=0] = 0 (12+ 8)

21. **(i)** State and prove Kolmogorov's strong law of large numbers.

(ii) State the conditions under which WLLN holds. (14+6)

22. State and prove the Lindeberg-Levi central limit theorem clearly explaining the assumptions.

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