LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECONDSEMESTER – APRIL 2017

16PST2MC01- ESTIMATION THEORY

Date: 19-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

SECTION – A

Answer ALL the questions (10 x 2 = 20)

1. Explain the problem of Point estimation.
2. State the different approaches to identify UMVUE
3. If δ is a UMVUE then show that δ + 2 is also a UMVUE.
4. Define Minimum Variance Bound Estimator.
5. Let X be random variable with pmf: $P(X=1)=\frac{\theta}{2}$, $P(X=2)=1-\frac{\theta}{2}$. Find the Fishers Information contained in X?
6. Let X_1, X_2 be iid P(θ) , θ>0. Show that X_1+2X_2 is not sufficient for θ.
7. Define Ancillary Statistic with an example.
8. Explain the concept of likelihood function.
9. Let X follow B(1, θ), θ = 0.1,0.2. Find MLE of θ .
10. Define CAN estimator.

SECTION – B

Answer any FIVE questions (5 x 8 = 40)

11. Given an example for each of the following:
(i) U_g is empty (ii) U_g is singleton.
12. State and Prove a necessary and sufficient condition for an estimator to be UMVUE using uncorrelatedness approach.
13. Let $X_1,X_2,\ldots,X_n$ be a random sample of size n from $U[0,\theta],\ \theta>0$. Find the Sufficient Statistic for θ .
14. Give an application of Basu's theorem.
15. Let $X_1, X_2,\ldots,X_n$ be a random sample of size n from $N(\mu,\sigma^2)$, $\mu\in\mathbf{R},\sigma>0$. Find UMVUE of $\frac{\mu^2}{\sigma^2}$.

16. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $P(\theta), \theta > 0$. Obtain MVBE of θ and suggest MVBE of $a\theta + b$, where a and b are constants such that $a \neq 0$.

17. MLE is not unique – Illustrate with an example.

18. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu,1),\ \mu \in R$. Let μ have the prior distribution $N(0,1)$. Find the Bayes estimator of μ.

SECTION – C

Answer any TWO questions **(2x 20 = 40)**

19. (a) If UMVUE exists for the parametric function $\psi(\theta)$ then show that it must be essentially unique.

(b) Let $X_1, X_2, ..., X_n$ be a random sample of size n from $N(\theta,1)$, $\theta \in R$. Obtain the Cramer- Rao lower bound for estimating θ^2. Compare the variance of the UMVUE with CRLB. (10+10)

20. (a) Explain completeness and boundedly completeness with an illustration.

(b) State and establish Lehmann-Scheffe theorem. (10+10)

21. (a) State and Prove Cramer-Rao inequality by stating its regularity conditions.

(b) Let $X_1, X_2, ..., X_n$ be a random sample of size n from $N(\mu, \sigma^2)$, $\mu \in R$, $\sigma^2 > 0$.

i) Show that $\bar{X}$ is sufficient statistic for μ when σ^2 known.

ii) Show that $(\bar{X}, S^2)'$ is sufficient statistic for (μ, σ^2) . (10+10)

22 (a) "Blind use of Jackknife method" – Illustrate with an example.

(b) Explain Baye's estimation with an example. (10+10)

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