LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRDSEMESTER – APRIL 2017

ST 3815- MULTIVARIATE ANALYSIS

Date: 06-05-2017 Dept. No. Max. : 100 Marks
01:00-04:00

SECTION – A

Answer ALL the questions (10 x 2 = 20)

1. Mention any two properties of multivariate normal distribution.
2. If $X' = (X_1, X_2) \sim N_2\left\{\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right\}$ then write the characteristic function of the marginal distribution of X_1.
3. In a multivariate normal distribution, show that every linear combination of the component variables of the random vector is normal. Is the converse true?
4. Define Hotelling's T^2 – Statistic. How is it related to Mahlanobis' D^2?
5. Comment on repeated measurements design.
6. Describe a) Common factor b) Communality.
7. Explain the classification problem into two classes.
8. Let $(X_i, Y_i)'$ i = 1,2,3 be independently distributed each according to $N_2\left\{\begin{pmatrix} \mu \\ \eta \end{pmatrix}, \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}\right\}$.

   Find the distribution of $(\bar{X}, \bar{Y})'$.
9. Explain MANOVA.
10. Write a short note on data mining.

PART– B

Answer anyFIVE questions (5X8=40 marks)

11. Find the multiple correlation coefficient between X_1 and $X_2, X_3, \ldots, X_p$. Prove that the conditional variance of X_1 given the rest of the variables cannot be greater than unconditional variance of X_1.
12. Derive the characteristic function of multivariate normal distribution.
13. Let $X_1, X_2, ..., X_n$ be independent $N(0, \sigma^2)$ random variables. Show that $(X' AX)/\sigma^2$ is chi-square if A is idempotent where $X = (X_1, X_2, ..., X_n)'$.
14. Obtain the linear function to allocate an object to one of the two given normal populations.

15. Using the Likelihood ratio procedure, develop the linear discriminant function and its variance.

16. Giving suitable examples explain how factor scores are used in data analysis.

17. What are principal components? Outline the procedure to extract principal components from a given correlation matrix.

18. Outline single linkage and complete linkage procedures with an example.

PART– C

Answer anyTWO questions (2 X 20 =40marks)

19. a) Derive the distribution function of the generalized T^2 – Statistic.

    b) Test $\mu=(0 \;\; 0)'$ at level 0.05, in a bivariate normal population with $\sigma_{11}=\sigma_{22}=10$ and $\sigma_{12}=-4$, using the sample mean vector $\bar{x}=(7 \;\; -3)'$ based on sample size 20. (15+5)

20. a) Define generalized variance.

    b) Show that the sample generalized variance is zero if and if the rows of the matrix of deviation are linearly dependent.

    c) Find the covariance matrix of the multinormal distribution which has the quadratic form $2x_1^{\,2}+x_2^{\,2}+4x_3^{\,2}-x_1x_2-2x_1x_3$. (3+12+5)

21. Consider the two data sets from populations Π_1 *and* Π_2 respectively,

$$X_1=\begin{pmatrix}3 & 7\\ 2 & 4\\ 4 & 7\end{pmatrix} \quad and \quad X_2=\begin{pmatrix}6 & 9\\ 5 & 7\\ 4 & 8\end{pmatrix}$$ for which

$$\bar{x}_1=(3\;6)' \;,\; \bar{x}_2=(5\;8)' \quad and \quad S_{pooled}=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$$

    a) Calculate the linear discriminant function.

    b) Classify the observation $x_0{}'=(2\;7)$ to population π_1 or population π_2 using the decision rule with equal priors and equal costs. (14+6)

22. a) Prove that the extraction of principal components from a dispersion matrix is the study of characteristic roots and vectors of the same matrix.

    b) Prove that under some assumptions (to be stated), variance covariance matrix can be written as $\Sigma = LL' + \Psi$ in the factor analysis model. Also discuss the effect of an orthogonal transformation. (8+12)

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