LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRDSEMESTER – APRIL 2017

ST 3816- STOCHASTIC PROCESS

Date: 26-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

SECTION - A

Answer ALL questions. **(10 x 2 = 20 marks)**

1. Define a stochastic process with an example.
2. When do you say a Markov chain is transient or recurrent?
3. When do you say the process $\{X_n\}$ is a Martingale?
4. Define a stationary process.
5. Write the PGF of the Poisson process.
6. Which of the following are true for a renewal process
   i) $N(t) \geq k$ if and only if $W_k \leq t$
   ii) $N(t) < k$ if and only if $W_k > t$?
7. Define a branching process.
8. Write the distribution of excess life for a Poisson renewal process.
9. State the Basic limit theorem on Markov chains.
10. Messages arrive at a telegraph office as a Poisson process with mean 3 messages per hour. What is the probability that no message arrive during 8.00 to 12.00 noon.

SECTION - B

Answer any FIVE questions. **(5 x 8= 40 marks)**

11. State and prove Chapman – Kolmogorov equations for a Markov chain.
12. Explain the discrete queueing Markov chain.
13. Determine the classes, periodicity of a Markov chain with states 0,1,2,3 and transition probability matrix

$$\text{matrix}\; \begin{matrix} & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 2 & 1/2 & 1/2 & 0 & 0 \\ 3 & 1/3 & 1/3 & 1/3 & 0 \end{matrix}$$

    And also find out whether they are recurrent or transient.
14. Write the postulates of pure birth process and obtain the differential equations. Solve for n = 0,1.
15. Prove any two properties of the Poisson process.
16. Define a renewal process. For a renewal process, show that $M(t) = F(t) + \int_0^t M(t-y)\, dF(y),\ t \geq 0$.
17. Show that for a branching process $\varphi_{n+1}(s) = \varphi_n[\varphi(s)]$.

18. For a two state process with transition probability matrix $P=\begin{bmatrix}1-a & a\\ b & 1-b\end{bmatrix}$, $0<a,b<1$.

Show that $P^n=\frac{1}{a+b}\begin{bmatrix}b & a\\ b & a\end{bmatrix}+\frac{(1-a-b)^n}{(a+b)}\begin{bmatrix}a & -a\\ -b & b\end{bmatrix}$. Obtain $\lim_{n\to\infty}P^n$.

SECTION – C

Answer any TWO questions. (2 x 20= 40 marks)

19. i) Prove that a state 'i' is recurrent if and only if $\sum_n P_{ii}^n=\infty$

ii) If $i\leftrightarrow j$ and if 'i' is recurrent. Prove that j is also recurrent.

iii) Discuss the recurrence of one dimensional random walk on the set of integers. **(8+5+7)**

20. a) State the postulates of the Birth and Death process. Obtain the forward and backward differential equations.

b) Using the forward differential equations, obtain $\lim_{t\to\infty}P_{ij}(t)$ **(15+5)**

21. a) Obtain the Expectation and Variance of the random sum $X_1+X_2+\ldots+X_N$, where X_i are iid, $E(X_i)=\mu$, $V(X_i)=\sigma^2$, $E(N)=V$ $V(N)=\tau^2$ and X_i are independent of N. Use it to find the mean and variance of the branching process.

b) Explain the probability of extinction for a branching process. Obtain the probability of extinction for the branching process with offspring distribution as $\begin{matrix}\xi & : & 0 & 2\\ P(\xi) & : & 1/3 & 2/3\end{matrix}$

(14+6)

22. a) Obtain the gamblers ruin (absorption into 0) by using first step analysis.

b) Consider a Markov chain with states 0,1,2,3,4,5 and transition probability matrix

$$\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 3/4 & 1/4 & 0 & 0 & 0\\ 0 & 1/8 & 7/8 & 0 & 0 & 0\\ 1/4 & 1/4 & 0 & 1/8 & 3/8 & 0\\ 1/3 & 0 & 1/6 & 1/6 & 1/3 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

Obtain $\lim_{n\to\infty}P_{3i}^n$, $i=0,1,2,3,4,5$ **(10+10)**

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