LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **MATHEMATICS**

THIRDSEMESTER – APRIL 2018

16UST3AL01- MATHEMATICAL STATISTICS - I

Date: 03-05-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION - A

Answer ALL questions: **(10X 2 = 20 Marks)**

1. Define: Statistics.
2. State Bayes theorem.
3. Write the additive property of Poisson distribution.
4. How will you derive the marginal density function from joint density function?
5. Write the MGF of Normal distribution.
6. Derive the mean of Geometric distribution.
7. What is the n^{th} order statistic?
8. Define: t statistic.
9. Write the expression for sample mean and sample variance.
10. What is convergence in probability?

SECTION - B

Answer any FIVE questions: **(5 X 8 = 40 Marks)**

11. (i) Define : a) Sample space b) Events c) Sure Event d) Impossible Event (1+1+1+1)Marks
    ii) State and prove the addition law of probability. (4 marks)
12. In an electronics laboratory, there are identically looking capacitors of three makes A_1, A_2 and A_3 in the ratio 2:3:4. It is known that 1% of A_1, 1.5% of A_2 and 2% of A_3 are defective. What percentage of capacitors in the laboratory are defective? If a capacitor picked at defective is found to be defective, what is the probability it is of make A_3 ?
13. State and prove Chebyshev's inequality.
14. Calculate the correlation co efficient for the following data.

X	43	21	25	42	57	59
Y	99	65	79	75	87	81

15. Prove that a linear combination of random variables $X_1, X_2, \ldots, X_n$ follow $N(\mu_i, \sigma_i^2)$ is also Normal.
16. Derive the Mean and variance of beta distribution of I kind.
17. Derive the probability density function of $X_{(1)}$ and $X_{(n)}$.
18. Derive the distribution function of F distribution.

SECTION - C

Answer any TWO questions **(2 X 20 = 40 Marks)**

19. (i) For random variables X and Y, the joint probability density function is given by (10 Marks)

$$f_{X,Y}(x,y) = \frac{1+xy}{4} \quad |x| \le 1,\ |y| \le 1$$
$$= 0 \quad \text{otherwise}$$

Find the marginal density $f_X(x), f_Y(y)$ and $f_{Y/X}(y/x)$. Are X and Y independent?

(ii) Let the probability density function of X_1 and of X_2 be given (10 Marks)

$$f(x_1, x_2) = \begin{cases} 2e^{-x_1 - 2x_2}, & x_1 > 0, x_2 > 0 \\ 0 \ otherwise \end{cases}$$

Now find the probability density of $Y = X_1 + X_2$.

20. (i) Derive the moment generating function of gamma distribution. (10 Marks)

(ii) State and prove the lack of memory property of exponential distribution. (10 Marks)

21. Derive the moment generating function of chi square distribution and hence derive the mean and variance.

22. State and prove the central limit theorem.

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