# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **APRIL 2018**

**16UST3MC02– ESTIMATING THEORY**

Date: 07-05-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

## PART A

Answer **ALL** the questions **(10X2=20 marks)**

1. Define unbiased estimator of a parametric function. Give an example.
2. Define Consistent Estimator.
3. Define Sufficient Statistic using the 'Information Approach'.
4. Define a Complete Family of distributions.
5. Describe the Method of Least Squares Estimation.
6. State the Least squares Estimator of $\beta_0$ in the model $Y = \beta_0 + \beta_1 X + \varepsilon$ .
7. What is the role of prior distribution?
8. Explain the Minimax Principle.
9. Describe Confidence Intervals.
10. State the 95% confidence interval for $\mu$ based on a random sample of size n from $N(\mu, 1)$.

## PART B

Answer any **FIVE** questions **(5 x 8 = 40 marks)**

11. State and Prove Cramer-Rao Inequality.
12. Define UMVUE and prove that it is unique.
13. Derive the moment estimators of the parameters of Gamma Distribution.
14. If $X_1, X_2, .., X_n$ is a random sample from $N(\mu, \sigma^2)$, with $\mu$ known, derive a sufficient statistic for $\sigma^2$ using Neyman Fisher Factorization Criterion.
15. If $X_1, X_2, ..,X_n$ is a random sample from Poisson ($\lambda$), show that $\sum_{i=1}^{n} X_i$ is a sufficient statistic by the conditional distribution approach.

16. Let $X_1, X_2, .., X_n$ be a random sample from $B(1, \theta)$, where the prior distribution of $\theta$ is Beta $(\alpha, \beta)$. Obtain the Bayes estimator of $\theta$.

17. Obtain $100(1 - \alpha)\%$ confidence interval for the variance of a normal distribution whose mean is also unknown.

18. Define 'Efficiency' of an estimator. Explain with an example.

## PART C

Answer any **TWO** questions **(2 x 20 = 40 marks)**

19. a) Define a consistent estimator. Obtain the same for the parameter 'p' of $B(1, p)$ based on a random sample of size 'n'.

    b) Show that the family of Poisson distributions $\{P(\lambda), \lambda > 0\}$ is complete. **(12 + 8)**

20. a) State and Prove Rao-Blackwell Theorem.

    b) Obtain the moment estimators of the parameters of $U(\theta_1, \theta_2)$ **(10 + 10)**

21. a) Derive the MLEs of the parameter $\theta$ based on a random sample from $U[\theta - 1, \theta + 1]$

    b) If the Cramer-Rao Lower bound is attained by an estimator of $\theta$, show that this estimator is the solution of the likelihood equation $\partial \operatorname{Log} L / \partial\theta = 0$ and is the maximum likelihood estimator of $\theta$ **(8 + 12)**

22. a) Discuss the Principle of Minimax and define a Minimax Estimator.

    b) Define prior and posterior distributions with suitable notations. Define Bayes Risk of an estimator and obtain two different expressions for it. **(6 + 14)**

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