LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRSTSEMESTER – APRIL 2018

17/16PST1MC03- STATISTICAL MATHEMATICS

Date: 28-04-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

SECTION – A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Prove that a divergent sequence may have a convergent subsequence.
2. Prove that the series 1 + 2 + 3 + ... + n + ... is divergent.
3. If L is the limit of the sequence (s_n), then prove that every open interval containing L contains all but a finite number of terms of the sequence.
4. Show that all subsequences of a convergent sequence of real numbers converge to the same limit.
5. Write the different formulae for differentiating the sum, difference, product and quotient of two functions.
6. State comparison test for the series of positive terms.
7. Show that the function $f(x) = x^2$ is unbounded on R but is bounded on each bounded interval of R.
8. Let $f(x) = |x|$ for $x \epsilon (-\infty, \infty)$. Prove that 'f' does not have a derivative at '0', even though 'f' is continuous at '0'.
9. Define upper and lower integral of a bounded function f over [a, b].
10. Explain linear independence of k vectors using an example.

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. Write the formula for s_n for the sequence 1, -4, 7, -10, 13, ... Verify if the following sequence is a subsequence of (s_n):

    (i) (1, 7, 13, ...)

    (ii) (-4, -10, -16, ...)

    (iii) (7, -4, 13, -10, ...)

    (iv) (4, 10, 16, ...).
12. State and establish Cauchy Criterion of Convergence of a series.
13. If the limit of a sequence of real numbers (s_n) exists, then prove that it is unique.
14. Let $\sum a_n$ be a series of non-negative numbers and let $s_n = a_1 + a_2 + ... + a_n$. Then show that

    (i) $\sum a_n$ converges if (s_n) is bounded.

(ii) $\sum a_n$ diverges if (s_n) is not bounded.

15. If $\sum a_n$ converges absolutely, then prove that the series $\sum a_n$ converges but not conversely.

16. Prove that each constant function f(x) = c is Riemann integrable on any interval [a, b] for any partition P of [a, b].

17. Let f(x) = x ($0 \leq x \leq 1$). Let σ be the partition $\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ of [0, 1]. Obtain U [f; σ] and L[f; σ].

18. Mention any four properties of the Riemann integral.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19(a). Show that a monotonic series converges if and only if it is bounded. (10)

19(b). Obtain $\lim_{n \to \infty} c^{1/n}$ where c is a fixed positive number. (10)

20. State and establish Leibnitz Rule on alternating series. (20)

21(a). Explain improper integral of the first, second and third kind and give an example for each kind. (8)

21(b). Check the convergence of the integrals: (i) $\int_1^\infty \frac{1}{x^2}\, dx$ (ii) $\int_0^\infty e^{-x}\, dx$ (iii) $\int_1^\infty \frac{1}{\sqrt{x}} dx$

(12)

22(a). State Taylor's formula and Maclaurin's Theorem with Lagranges form of remainder.

Hence write Taylor's formula for f(x) = log(1 + x) about a = 2 and n = 4. (10)

22(b). Explain the inductive procedure of Gram-Schmidt Orthogonalization. (10)

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