LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **STATISTICS**

SECONDSEMESTER – APRIL 2018

17PST2ES02- MODERN PROBABILITY THEORY

Date: 25-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Section A

Answer ALL questions(2 X 10 = 20)

1. Define probability measure.
2. Define measurable function.
3. Define Lebesgue-Stieltjes measure.
4. What is meant by independence of classes?
5. Define mixture of distribution functions.
6. Define independence of n random variables.
7. State Basic Inequality.
8. Define bivariate characteristic function.
9. Define convergence in distribution.
10. State Chebyshev's WLLN.

Section B

Answer ANY FIVE questions. (5 X 8 = 40)

11. Define a Borel sigma filed and show that any interval is a Borel set, but the converse is not true.

12. Let F be the distribution function on R given by,

$$F(x)=\begin{cases} o, \; if \;\; x<-1 \\ 1+x, \; if \; -1\le x<0 \\ 2+x^2, \; if \;\; 0\le x<2 \\ 9, \quad if \; x\ge 2 \end{cases}$$

If µ is a Lebesgue-Stieltjes measure corresponding to F, compute the measure of each of the following sets. (i) {2}, (ii) [-1/2, 3), (iii) (-1, 0]U(1, 2), (iv) [0,1/2)U(1,2].

13. State and prove the continuity property of probability.

14. State and prove (i) Minkowski Inquality and (ii) Jensen's Inequality

15. State the inversion theorem for discrete and continuous case and find the distribution if the c.f is $\varphi(u)=e^{-|t|}$, $-\infty<t<\infty$.

16. Define convergence in probability and show that convergence in probability implies convergence in distribution.

17. State and prove weak law of large numbers for the iid case.

18. Show that convergence in probability implies convergence in r^{th} mean.

Section C

Answer ANY TWO questions. **(2 X 20 = 40)**

19. (i) Define independent events, independent classes, independent random variables.

(ii) State and prove the two necessary and sufficient condition for n random variables to be independent.

(6 + 14)

20. (i) Show that convergence in probability implies convergence in distribution.

(ii) Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$. Then

(a) $a X_n \xrightarrow{P} a X$

(b) $X_n + Y_n \xrightarrow{P} X + Y$

(c) $X_n Y_n \xrightarrow{P} X Y$

(d) $\frac{X_n}{Y_n} \xrightarrow{P} \frac{X}{Y}$ (12+ 8)

21. (i) State and prove Kolmogorov's strong law of large numbers.

(ii) Prove Chebyshev's WLLN. (10+ 10)

22. State and prove the Lindeberg-Levi central limit theorem clearly explaining the assumptions.

$$$$$$$$$