LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECONDSEMESTER – APRIL 2018

17/16/ ST2814 PST2MC01- ESTIMATION THEORY

Date: 17-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION – A

Answer ALL the questions **(10 x 2 = 20)**

1. If X_1 and X_2 are $B(1,\theta)$ then obtain the unbiased estimator of 2θ.
2. State the different approaches to identify UMVUE.
3. Define Minimum Variance Bound Estimator.
4. State Neyman - Fisher Factorization Theorem
5. Let X_1 and X_2 be iid $N(\theta,1)$, $\theta \in R$. Is $X_1 + 2X_2$ ancillary statistic?
6. Let X_1, X_2 be iidP(θ) , θ>0. Show that X_1+2X_2 is not sufficient for θ.
7. Define completeness and bounded completeness.
8. What is exponential class of family?
9. Suggest an MLEforP[X=0]in the caseofP(θ), θ>0.
10. Define CAN estimator.

SECTION – B

Answer any FIVE questions **(5 x 8 = 40)**

11. State and establish Uncorrelatedness approach of UMVUE.
12. Give an example for each of the following:
    (i) U_g is empty (ii) U_g is singleton.
13. State and establish Rao-Blackwell theorem.
14. Let $X_1,X_2,\ldots,X_n$ be a random sample from $N(0,\theta^2)$. Obtain the Cramer – Rao lower bound for estimating θ^2.
15. Show that the family of $B(n,\theta)$, $0<\theta<1$ is complete.
16. Let $X_1,X_2,\ldots,X_n$ be a random sample of size n from $P(\theta)$, $\theta>0$. Obtain MVBE of θ and suggest MVBE of $a\theta+b$, where a and b are constants such that $a \neq 0$.
17. State and establish Basu's theorem.
18. Let $X \sim N(0,\theta), \theta>0$. Assume that the prior distribution of θ is $h(\theta)=\theta e^{-\theta}, \theta>0$. Find the Baye's estimator of θ if the loss function is absolute error.

SECTION – C

Answer any TWO questions **(2x 20 = 40)**

19. (a) If UMVUE exists for the parametric function $\psi(\theta)$ then show that it must be essentially unique.

(b) Let $X_1, X_2,..., X_n$ be a random sample of size n from $N(\theta,1)$, $\theta \in R$.

i. Obtain the information contained in the sample.

ii. Show that $\overline{X}$ is MVBE for estimating θ.

iii. Deduce that $\overline{X}$ is UMVUE for estimating θ. (10+10)

20. (a) Give an example of an estimator which is consistent but not CAN.

(b) Let $X_1, X_2,..., X_n$ be a random sample of size n from a two parameter exponential distribution $E(\xi,\tau)$, $\xi \in R, \quad \tau > 0.$ Find MLE of $\xi \, and \, \tau$. (10+10)

21. (a) State and Prove Cramer-Rao inequality by stating its regularity conditions.

(b) MLE is not consistent – Support the statement with an example. (10+10)

22. (a) “Blind use of Jackknifed method” – Illustrate with an example.

(b) Write a short note on Bootstrap method. (10+10)

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