LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECONDSEMESTER – APRIL 2018

17/16PST2MC02/ST2815 - TESTING STATISTICAL HYPOTHESES

Date: 19-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Part – A

Answer ALL the questions **10x2=20 Marks**

1. Define Statistical Hypothesis with an example

2. What is Randomized test?

3. Define power function and Type I error

4. Define Level of Significance

5. Define Unbiased test

6. State any two properties of Likelihood ratio Statistic

7. Give an example of Scale invariant statistic

8. Define Uniformly Most Powerful Test

9. i) State the use of NP Lemma

ii) State the use of MLR property

10. Define p-value and provide any one use of p-value

Part – B

Answer any FIVE questions **5X8=40 Marks**

11. Let X ~ B(n,p) with n=5. To test H_0:p=1/4 vs H_1:p=1/2 the test function is defined as

$$\phi(x)=\begin{cases}0.3, \; if \;\; x=0 \\ 0.2, \; if \;\; x=1 \\ 0, \; otherwise\end{cases}$$

Determine Type I error, Type II error and Power of the test

12. Use Linear programming approach to find the Most powerful test for testing H:θ=0.1 vs

K:θ=0.2 based on X where X~B(1,θ), θ=0.1,0.2 Find MP test for level i) α=0.05 ii) α=0.1

13. Let $X_1,X_2,X_3,\ldots X_n$ be a random sample from N(θ,1). Obtain UMP level α test for testing

H_0:θ≤θ_0 against H_1:θ>θ_0 show that the test is consistent

14. Obtain Likelihood ratio test for testing H:μ=μ_0 against K: μ≠μ_0 based on random sample of size n from Normal distribution with unknown variance.

15. Let $\underline{x}$ have the pdf $P_\theta(x), \theta \in \Theta$. If $\{P_\theta(x), \theta \in \Theta\}$ posses MLR property in $T(\underline{x})$

For testing $H_0 : \theta \le \theta_0, H_1 : \theta > \theta_0$ show that the UMP level α test given by

$$\phi^*(x) = \begin{cases} 1, & T(\underline{x}) > c \\ \gamma, & T(\underline{x}) = c \\ 0, & T(\underline{x}) < c \end{cases}$$ where c and γ are determined such that $E_{\theta_0}(\phi^*) = \alpha$

16. Let $X_1, X_2, X_3, \ldots X_n$ be a random sample from N(μ,σ^2) where σ^2 is known. Obtain UMPU level α test for testing H:μ=μ_0 against K: μ≠μ_0 (Apply UMPU test for one parameter exponential family)

17. Find UMP invariant test for testing $H_0 : \sigma^2 \le \sigma_0^{\ 2}, H_1 : \sigma^2 > \sigma_0^{\ 2}$ based on a random sample of size n from normal distribution with unknown mean.

18. If L(x) is the Likelihood ratio for testing H:θ=θ_0 against K:θ≠θ_0 then show that the asymptotic distribution of -2L(X) is $\chi^2_{(1)}$

Part – C

Answer any TWO questions **2X20=40Marks**

19. i) State and Prove Neyman Pearson Lemma based on Non-Randomized test function(15)

ii) State Neyman Pearson Lemma based on Randomized test (Proof not required) (5)

20. Obtain Likelihood Ratio test for testing H: μ_1=μ_2 against K: μ_1≠μ_2 based on random sample of size n from normal distribution with equal variance (variance unknown)

21. i) Let $X_1, X_2, .., X_n$ be a random sample from U[0,θ], θ>0 show that UMP level α test for testing H:θ≤θ_0 against K:θ>θ_0 and also obtain the power function.

ii) Compare the test function $\phi_2(\underline{x}) = \begin{cases} 1, & x_{(n)} > \theta_0 \\ \alpha, & x_{(n)} < \theta_0 \end{cases}$ with UMP obtained in (i) and hence

~~show that $\phi_2(x)$ also has the same power function in the alternative test~~

22. i) Define Multiparamter Exponential Family

ii) State the UMPU test for (k+1) – parameter exponential family for testing

$H: \theta \leq \theta_0$ vs $K: \theta > \theta_0$

iii) Let $X_1, X_2, X_3, \ldots X_n$ be a random sample from $N(\mu, \sigma^2)$ Use UMPU defined in (ii) and obtain the level α test for testing $H_0 : \sigma^2 \leq \sigma_0^2, H_1 : \sigma^2 > \sigma_0^2$

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