LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECONDSEMESTER – APRIL 2018

17/16PST2MC03/ST2816 - SAMPLING THEORY

Date: 21-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

SECTION – A

Answer ALL the questions. (10 x 2 = 20 marks)

1. Define Probability Sampling Design. Explain its two types.
2. What is meant by first and second order inclusion probabilities?
3. State and prove a necessary and sufficient condition for the existence of an unbiased estimator for population total.
4. Define an unbiased estimator for population total under PPSWR and show that it is unbiased.
5. Describe Random Group Method of sampling and define an estimator for population total under this method.
6. In LSS, list all possible samples of size '4', from a population of size '20'.
7. In BSS, when the population is linear, show that the expansion estimator equals the population total.
8. Prove that the Desraj ordered estimator is unbiased for the population total.
9. In SRSWOR, show that $E_P(s_{xy}) = S_{xy}$.
10. Explain 'Multistage Sampling'.

SECTION - B

Answer any FIVE questions. (5 x 8 = 40 marks)

11. For any sampling design, prove that

(i) $\sum_{j=1}^{n} \pi_{ij} = (n-1)\,\pi_i; \quad i = 1, 2, \ldots, N; \quad j \neq i$

(ii) $\sum_{j=1}^{n} (\pi_i \pi_j - \pi_{ij}) = \pi_i (1 - \pi_i); \quad i = 1, 2, \ldots, N; \quad j \neq i$

12. For any design, obtain V ($\hat{Y}_{HT}$).
13. Check if v ($\hat{Y}_{HT}$) ≥ 0 for all 's' receiving positive probabilities under MSD.
14. In LSS, under linear population, obtain Yates' corrected estimator for estimating population total without error.
15. For any sample of size 'n', obtain v($\hat{Y}_{DR}$).
16. Explain proportional allocation in Stratified Sampling and deduce V($\hat{Y}_{St}$) under this allocation.
17. Explain Simmons' unrelated randomized response method for estimating π_A when π_Y is known.

18. Derive the approximate expression for B($\hat{Y}_R$) and MSE($\hat{Y}_R$).

SECTION – C

Answer any TWO questions. (2 x 20 = 40 marks)

19(a) After the decision to take a SRS has been made, it was realized that Y_1, the value of population unit 1 would be unusually low and Y_N, the value of population unit N would be unusually high. In such cases, it is decided to use the estimator

$$\frac{\hat{}}{Y}^* = \frac{\hat{}}{Y} + c \quad \text{if} \quad 1 \in s,\ N \notin s,$$

$$\frac{\hat{}}{Y}^* = \frac{\hat{}}{Y} - c \quad \text{if} \quad 1 \notin s,\ N \in s, \text{ and}$$

$$\frac{\hat{}}{Y}^* = \frac{\hat{}}{Y} \quad \text{otherwise,}$$

where 'c' is a predetermined constant. Show that $\frac{\hat{}}{Y}^*$ is unbiased for $\bar{Y}$ for any 'c'.

Derive V ($\frac{\hat{}}{Y}^*$). Find the value of 'c' for which $\frac{\hat{}}{Y}^*$ is more efficient than $\frac{\hat{}}{Y}$. (12)

(b) Deduce the formula for $\hat{Y}_{St}$, V($\hat{Y}_{St}$) and v($\hat{Y}_{St}$) when samples are drawn independently from different strata using (i) SRSWOR and (ii) PPSWR. (8)

20(a) Suppose from a sample of n units selected using SRS, a subsample of n' units is selected using SRS and included in the original sample. Obtain the expected value and the approximate sampling variance of $\hat{\bar{y}}'$, the sample mean based on (n + n') units. (10)

(b) Obtain the expression for Π_i and Π_{ij}, under MSD. (10)

21(a) Describe Regression Estimation and derive an approximate expression for the bias and MSE of $\hat{Y}_{LR}$. (10)

(b) A SRS of size $n = n_1 + n_2$ with mean $\frac{\hat{}}{Y}$ is drawn from a finite population of N units and a SR subsample of size n_1 is drawn from it with mean $\frac{\hat{}}{Y_1}$. Derive $V_P(\frac{\hat{}}{Y_1} - \frac{\hat{}}{Y_2})$, where $\frac{\hat{}}{Y_2}$ is the mean of the remaining n_2 units in the sample. (10)

22(a) Verify if the Hansen-Hurwitz estimator $\hat{Y}_{dhh}$ under double sampling is unbiased for Y and find V($\hat{Y}_{dhh}$). (10)

(b) In Two – Stage Sampling with SRS in both stages, obtain the mean and variance of the estimator $\hat{Y}_{TS}$, for estimating population total. (10)

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