

# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



## M.Sc.DEGREE EXAMINATION –STATISTICS

FIRST SEMESTER – APRIL 2018

17/16PST1MC01 / ST 1815 / ST 1820– ADVANCED DISTRIBUTION THEORY

Date: 25-04-2018

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

### SECTION – A

Answer ALL the questions

(10 x 2 = 20 Marks)

1. Write the pdf of Truncated Binomial distribution truncated at 0.
2. Define power series distribution.
3. Write the MGF of trinomial distribution.
4. Let  $X$  follow exponential distribution with parameter  $\alpha$ . Write the distribution of the first order statistic when a random sample of size  $n$  is drawn.
5. Let  $X_1, X_2, \dots, X_n$  be iid from  $\text{Poisson}(\theta)$ . Let the prior distribution be  $h(\theta) = e^{-\theta}, \theta > 0$ . Obtain the posterior density  $g(\theta | x)$ .
6. Define a quadratic form in  $n$  variables.
7. Define: PGF. Write the PGF of a Poisson distribution.
8. Write any 4 properties of a distribution function.
9. Define non-central F-Statistic.
10. Write the distribution of sample mean and  $\sum (X_i - \bar{X})^2 / \sigma^2$  for a random sample from  $N(\mu, \sigma^2)$ .

### SECTION – B

Answer any FIVE questions

(5 x 8 = 40 Marks)

11. Obtain the mean, median and mode of lognormal distribution.
12. Show that Geometric distribution satisfies lack of memory property.

13. For the distribution function  $F(x) = \begin{cases} 0 & x < -1 \\ \frac{x+2}{4} & -1 \leq x \leq 1 \\ 1 & 1 \leq x < \infty \end{cases}$

Obtain the decomposition of  $F$ . Find the mean and variance.

14. Obtain the recurrence relation satisfied by the power series distribution.
15. Obtain the PGF of Bivariate Poisson distribution. Hence obtain the marginal distributions.
16. Let  $X_1 \sim G(\alpha, p_1)$  and  $X_2 \sim G(\alpha, p_2)$ .  $X_1$  and  $X_2$  are independent. Obtain the pdf of  $\frac{X_1}{X_1 + X_2}$ .

17. Derive the MGF of Bivariate Normal distribution.

18. Explain compound distributions in detail.

### SECTION – C

Answer any TWO questions

(2 x 20 = 40 Marks)

19. Derive the pdf of Bivariate Binomial distribution. Obtain the PGF. Hence, obtain the marginal distributions of  $X_1$  and  $X_2$ .

Show that  $X_1$  given  $X_2 = x_2$  is equal in distribution to  $U_1 + V_1$  where  $U_1$  and  $U_2$  have Binomial distributions and  $U_1$  &  $V_1$  are independent. Hence obtain the correlation between  $X_1$  and  $X_2$ .

20. a) State and Prove Skitovitch theorem. (10)

b) Obtain the characterization of Normal distribution through the independence of  $X_1 + X_2$  and  $X_1 - X_2$ . (10)

21. Obtain the three different characterizations of Exponential distribution.

22. Derive the pdf of non-central t-distribution.

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