LOYOLA COLLEGE (AUTONOMOUS), CHENNAI -600 034

B. Sc. DEGREE EXAMINATION – STATISTICS

THIRD SEMESTER – NOV 2017

16UST3MC02 – ESTIMATION THEORY

DATE: **MAX. :100 Marks**

TIME:

PART A

Answer **ALL** the questions **(10 X 2=20 marks)**

1. Define unbiased estimator of a parametric function. Give an example.
2. Define Consistent Estimator.
3. Define Sufficient Statistic using the 'Information Approach'.
4. Define a Complete Family of distributions.
5. Describe the Method of Least Squares Estimation.
6. State the Least squares Estimator of β_0 in the model $Y = \beta_0 + \beta_1 X + \varepsilon$.
7. What is the role of prior distribution?
8. Explain the Minimax Principle.
9. Describe Confidence Intervals.
10. State the 95% confidence interval for μ based on a random sample of size n from $N(\mu, 1)$.

PART B

Answer any **FIVE** questions **(5 x 8 = 40 marks)**

11. State and Prove Cramer-Rao Inequality.
12. Define UMVUE and prove that it is unique.
13. Derive the moment estimators of the parameters of Gamma Distribution.
14. If $X_1, X_2, .., X_n$ is a random sample from $N(\mu, \sigma^2)$, with μ known, derive a sufficient statistic for σ^2 using Neyman Fisher Factorization Criterion.
15. If $X_1, X_2, .., X_n$ is a random sample from Poisson (λ), show that $\sum_{i=1}^{n} X_i$ is a sufficient statistic by the conditional distribution approach.
16. Let $X_1, X_2, .., X_n$ be a random sample from $B(1, \theta)$, where the prior distribution of θ is Beta (α, β). Obtain the Bayes estimator of θ.

17. Obtain $100(1 - \alpha)\%$ confidence interval for the variance of a normal distribution whose mean is also unknown.

18. Define 'Efficiency' of an estimator. Explain with an example.

PART C

Answer any **TWO** questions **(2 x 20 = 40 marks)**

19. a) Define a consistent estimator. Obtain the same for the parameter 'p' of B(1, p) based on a random sample of size 'n'.

    b) Show that the family of Poisson distributions $\{P(\lambda), \lambda > 0\}$ is complete. (12 + 8)

20. a) State and Prove Rao-Blackwell Theorem.

    b) Obtain the moment estimators of the parameters of $U(\theta_1, \theta_2)$ (10 + 10)

21. a) Derive the MLEs of the parameter θ based on a random sample from $U[\theta - 1, \theta + 1]$

    b) If the Cramer-Rao Lower bound is attained by an estimator of θ, show that this estimator is the solution of the likelihood equation $\partial \text{ Log } L / \partial\theta = 0$ and is the maximum likelihood estimator of θ (8 + 12)

22. a) Discuss the Principle of Minimax and define a Minimax Estimator.

    b) Define prior and posterior distributions with suitable notations. Define Bayes Risk of an estimator and obtain two different expressions for it. (6 + 14)

* * * * * * *