LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECONDSEMESTER – APRIL 2018

ST 2962- MODERN PROBABILITY THEORY

Date: 27-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Section A

Answer ALL questions(2 X 10 = 20)

1. Define Borel sigma field.
2. Define probability measure.
3. Define a random variable.
4. Define Lebesgue measure.
5. Define independence of n random variables.
6. Define mixture of distribution functions.
7. State Basic Inequality.
8. Define characteristic function.
9. Define convergence in probability.
10. State Lindeberg-Feller Central Limit theorem.

Section B

Answer ANY FIVE questions(5 X 8 = 40)

11. State and prove the continuity property of probability.
12. State and prove the necessary and sufficient condition for n random variables to be independent.
13. Define expectation of discrete and continuous random variables and statewith proof all the properties of expectation of a random variable.
14. Prove Basic inquality.
15. Show that convergence in probability implies convergence in distribution.
16. Show that convergence in r^{th} mean implies convergence in probability.
17. State and prove Kolmogorov's strong law of large numbers.
18. State and prove the Chebyshev's WLLN.

Section C

Answer ANY TWO questions. (2 X 20 = 40)

19. (i) State and prove the necessary and sufficient condition for a function F to be the distribution function of a random variable.

(ii)Let F be the distribution function on R given by,

$$F(x)=\begin{cases} o, \ if \ \ x<-1 \\ 1+x, \ if \ \ -1\le x<0 \\ 2+x^2, \ if \ \ 0\le x<2 \\ 9, \quad if \ x\ge 2 \end{cases}$$

If μ is a Lebesgue-Stieltjes measure corresponding to F, compute the measure of each of the following sets. (i) {2}, (ii) [-1/2, 3), (iii) (-1, 0]U(1, 2), (iv) [0,1/2)U(1,2]. (12+8)

20. (i) Show that the characteristic function of a random variable is continuous.

(ii) State the inversion theorem for discrete and continuous case

(iii) Derive characteristic function of normal distribution

(iv) Find the distribution function if the characteristic function is

$\varphi(u)=e^{-|t|}$, $-\infty<t<\infty$ (5+5+5+5)

21. State and prove weak law of large numbers for the iid case and non iid case.

22. State and prove the Lindeberg-Levi central limit theorem clearly explaining the assumptions.

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