LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION –**STATISTICS**

THIRD SEMESTER – **APRIL 2018**

ST 3506– MATRIX AND LINEAR ALGEBRA

Date: 07-05-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Part – A

Answer ALL the Questions **(10 x 2 = 20 marks)**

1) Define skew symmetric matrix with an example.
2) Define minor of an element in a matrix. Find the minor of a_{21} in the following matrix
$$\begin{bmatrix} 2 & 5 & 8 \\ 1 & 3 & 2 \\ 0 & 4 & 6 \end{bmatrix}$$
3) Mention any two properties of the determinant of a matrix.
4) Find the value $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{vmatrix}$
5) Establish the uniqueness of the inverse of a non-singular matrix.
6) Find the rank of $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$.
7) Define Linear Independence of vectors.
8) Define a linear transformation.
9) Define the characteristic equation of amtrix.
10) Find the characteristic roots of $\begin{bmatrix} 5 & 1 \\ 2 & 4 \end{bmatrix}$.

Part – B

Answer any FIVE Questions **(5 x 8 = 40 marks)**

11) If matrices A and B commute in multiplication, show that
$(A+B)^n = \sum_{i=1}^{n} {}^nC_i \, A^i \, B^{n-i}$ for every positive integer 'n'.
12) Show that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix.
13) Find the inverse of the following matrix by step-by-step reduction of [A ⋮ I]:
$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

14) (a) Show that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)\,(b-c)\,(c-a)$.

(b) Show that $\begin{vmatrix} x+2 & 3 & 4 \\ 2 & x+3 & 4 \\ 2 & 3 & x+4 \end{vmatrix} = x^2(x+9)$ (4 + 4)

15) State and prove a necessary and sufficient condition for a square matrix to possess an inverse.

16) If the coefficient matrix A is non-singular, show that the system Ax = 0 of n homogeneous linear equations in n unknowns has only a trivial solution.

17) In a vector space of dimension 'n', show that any vector x is a linear combination of linearly independent vectors $x_1, x_2, \ldots, x_k$ if and only if $x, x_1, x_2, \ldots, x_k$ are linearly dependent.

18) Show that the product of two matrices is associated to composition of linear transformations.

Part – C

Answer any TWO Questions **(2 x 20 = 40 marks)**

19) a) Establish that Tr(AB) = Tr(BA) with the following matrices:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$$

b) Applying the properties of a determinant, find the value of the following determinant:

$$\begin{vmatrix} 2 & 1 & -2 & -1 & 3 \\ 1 & 2 & -1 & 4 & 1 \\ -2 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 2 & 3 \\ 1 & 1 & 0 & 3 & 1 \end{vmatrix}$$ **(10 + 10)**

20) a) Obtain the expression for the inverse of a matrix in partitioned form.

b) Find inverse of the following matrix by partitioning:

$$\begin{bmatrix} 0 & 1 & 1 \\ 2 & 3 & -1 \\ -1 & 2 & 1 \end{bmatrix}$$ **(10 + 10)**

21) a) State and prove Cayley-Hamilton Theorem.

b) Show that the following matrix satisfies Cayley-Hamilton Theorem:

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$ **(10 + 10)**

22) a) Show that the characteristic roots of two similar matrices are the same.

b) Using Cramer's rule to solve the following system of equations:

$2x_1 + 3x_2 + x_3 - x_4 = 1$, $x_1 + x_2 - x_4 = -1$

$3x_2 + x_3 + x_4 = 3$, $x_1 + x_3 - x_4 = 0$ **(5 + 15)**

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