## ST 4209/ST 4206/ST 4201- MATHEMATICAL STATISTICS

## SECTION - A

## Answer ALL questions:

(10X 2 = 20 Marks)

1. Define: Statistics.
2. What is conditional distribution?
3. Write the additive property of Binomial distribution.
4. How will you derive the marginal density function from joint density function?
5. Write the MGF of Poisson distribution.
6. Derive the mean of Exponential distribution.
7. What is the $\mathrm{n}^{\text {th }}$ order statistic?
8. Define: $t$ statistic.
9. Define: unbiased estimator.
10. Define: Type II error.

## SECTION - B

## Answer any FIVE questions:

11. State and prove the addition law of probability.
12. If the joint pdf of $(\mathrm{X}, \mathrm{Y})$ is given by $f(x, y)=e^{-(x+y)}, x \geq 0, y \geq 0$. Find $\mathrm{E}(\mathrm{XY})$.
13. State and prove Chebyshev's inequality.
14. Calculate the correlation co efficient for the following data.

| X | 43 | 21 | 25 | 42 | 57 | 59 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 99 | 65 | 79 | 75 | 87 | 81 |

15. Prove that a linear combination of random variables $X_{1}, X_{2}, \ldots, X_{n}$ follow $N\left(\mu_{i}, \sigma_{i}^{2}\right)$ is also Normal.
16. Derive the Mean and variance of Discrete Uniform distribution.
17. A random sample ( $\left.X_{1}, X_{2}, X_{3}, X_{4}, X_{5}\right)$ of size 5 is drawn from normal population with unknown mean $\mu$. Consider the following estimators.
i) $t_{1}=\frac{X_{1}+X_{2}+X_{3}+X_{4}+X_{5}}{5}$, ii) $t_{2}=\frac{X_{1}+X_{2}}{2}+X_{3}$ iii) $t_{3}=\frac{2 X_{1}+X_{2}+\lambda X_{3}}{3}$

Find $\lambda$. Are $t_{1}$ and $t_{2}$ unbiased? State giving reasons, the estimator which is best among $t_{1}, t_{2}$ and $t_{3}$ ? 18. Define the following:
(i) Null Hypothesis
(ii) Alternate Hypothesis
(iii) Critical region (iv) Most Powerful critical region

## SECTION - C

## Answer any TWO questions

19. Two random variables X and Y have the joint $\operatorname{pdf} f(x, y)=\left\{\begin{array}{c}\frac{x y}{96}, 0<x<4,1<y<5 \\ 0\end{array}\right.$ otherwise. . Find (i) $\mathrm{E}(\mathrm{X})$
(ii) $\mathrm{E}(\mathrm{Y})$ (iii) $\operatorname{Var}(\mathrm{X}$
(iv) $\operatorname{Var}(\mathrm{Y})$
(v) $\mathrm{E}(\mathrm{XY})$
(vi) $\mathrm{E}(2 \mathrm{X}+3 \mathrm{Y})$ (vii) $\operatorname{COV}(\mathrm{X}, \mathrm{Y})$.
20. (i) Derive the moment generating function of Normal distribution.
(ii) State and prove the lack of memory property of exponential distribution.
21. Derive the moment generating function of chi square distribution and hence derive the mean and variance.
22. State and prove Neyman - Pearson Lemma.
