LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FOURTH & FIFTH SEMESTER – **APRIL 2018**

ST 4503 / ST 5504 – ESTIMATION THEORY

Date: 08-05-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART A

Answer **ALL** the questions: **(10X2=20 marks)**

1. Distinguish between point estimation and interval estimation.
2. Define unbiased estimator. Give example.
3. Explain sufficient estimator.
4. Let X_1, X_2 denote a random sample from Poisson distribution with parameter λ. Obtain sufficient statistics.
5. State any two properties of Maximum Likelihood Estimators.
6. Describe the method of least square estimation.
7. Explain Posterior distribution.
8. What is Baye's estimator?
9. Describe the general method of constructing confidence interval.
10. Obtain $100(1-\alpha)\%$ confidence interval for the variance, if a random sample of size n is drawn from normal population with known mean.

PART B

Answer any **FIVE** questions: **(5X8=40 marks)**

11. Obtain any two unbiased estimators for the population mean in the case of normal distribution.
12. Show that minimum variance unbiased estimator is unique.
13. Let $X_1, X_2, ..., X_n$ be a random sample of size n from $N(\mu, \sigma^2); \mu \in R,\ \sigma > 0$. Find sufficient estimators of μ and σ^2.
14. State and prove Lehmann-Scheffe theorem.
15. Describe method of moment estimation.

16. Distinguish between prior and posterior distribution.

17. Define Loss function and Risk function with an example for each.

18. Obtain the MLE of λ in the case of Poisson distribution based on a random sample of size n.

PART C

Answer any **TWO** questions: **(2X20=40 marks)**

19. a. State and prove Cramer-Rao inequality.

    b. Obtain the Cramer – Rao lower bound for the unbiased estimator of λ in the case of Poisson distribution

20. a. State and establish Rao Blackwell theorem.

    b. If $X_1, X_2, ..., X_n$ is a random sample from Bernoulli distribution

$$f(x;\theta) = \begin{cases} \theta^x (1-\theta)^{1-x} & x = 0, 1, \; 0 < \theta < 1 \\ o.w. \end{cases}$$

Then show that $\sum_{i=1}^{n} X_i$ is complete for θ.

21. a. Explain the method of maximum likelihood.

    b. Let $X_1, X_2, ..., X_n$ be a random sample from $f(x;\theta) = \begin{cases} \theta\, e^{-\theta x} & x > 0, \theta > 0 \\ 0 & o.w. \end{cases}$

    Estimate θ using method of moments.

22. a. Construct 100 $(1-\alpha)\%$ confidence interval for themeans of a independent normal population whose variance is known.

b. Let $X \sim b(n, p)$ and $L(p, \delta(x)) = (p - \delta(x))^2$. Let $\pi(p) = 1$ for $o < p < 1$ be the prior PDF of p. Find Bayes estimator.

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