LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034
M.Sc.DEGREE EXAMINATION - STATISTICS

FOURTHSEMESTER - APRIL 2018

## ST 4811/ST 4814-ADVANCED OPERATIONS RESEARCH

Date: 27-04-2018
Dept. No. $\square$ Max. : 100 Marks

## SECTION A

Answer ALL questions. Each carries two marks.

1. Define a General Linear Programming Problem.
2. Define duality..
3. What are the applications of dynamic programming?
4. State the principle of optimality in dynamic programming.
5. What is integer programming problem?
6. Define Non Linear Programming Problem.
7. Define a quadratic programming problem.
8. What is inventory control?
9. What are the costs associated with inventory?
10. What is a queuing system?

## SECTION B

Answer any FIVE questions. Each carries eight marks.
11. Use two-phase simplex method to maximize $Z=3 X_{1}+2 X_{2}$,
subject to the constraints, $2 \mathrm{X}_{1}+\mathrm{X}_{2} \leq 2 ; 3 \mathrm{X}_{1}+4 \mathrm{X}_{2} \geq 12$;and $\mathrm{X}_{1}, \mathrm{X}_{2} \geq 0$.
12. Derive Gomory's constraint for solving a Mixed Integer Programming Problem.
13. State the necessary conditions for solving the following Quadratic programming Problem.

Max $Z=6 X_{1}+3 X_{2}-4 X_{1} X_{2}-2 X_{1}^{2}-3 X_{2}^{2}$ subject to the constraints,
$X_{1}+X_{2} \leq 1 ; 2 X_{1}+3 X_{2} \leq 4$; and $X_{1}, X_{2} \geq 0$, and show that $Z$ is strictly concave.
14. Test for extreme values of $f\left(x_{1}, x_{2}, x_{3}\right)=x_{1}^{2}+x_{2}^{2}+x_{3}^{2}$, subject to the constraints,
$x_{1}+x_{2}+3 x_{3}=2$ and $5 x_{1}+2 x_{2}+x_{3}=5$.
15. Using Dynamic Programming Problem, maximize $z=y_{1} \cdot y_{2} \cdot y_{3}$ subject to the constraints,
$\mathrm{y}_{1}+\mathrm{y}_{2}+\mathrm{y}_{3}=15$, and $\mathrm{y}_{\mathrm{j}} \geq 0$.
16. A corporation is entertaining proposals from its 3 plants for possible expansion of its facilities. The corporation's budget is $£ 5$ millions for allocation to all 3 plants. Each plant is requested to submit its proposals giving total cost C and total revenue R for each proposal. The following table summarizes the cost
and revenue in millions of pounds. The zero cost proposals are introduced to allow for the probability of not allocating funds to individual plants. The goal of the corporation is to maximize the total revenue resulting from the allocation of $£ 5$ millions to the three plants.

|  | Plant 1 |  |  | Plant 2 |  | Plant 3 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Proposal | $\mathrm{C}_{1}$ | $\mathrm{R}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{R}_{2}$ | $\mathrm{C}_{3}$ | $\mathrm{R}_{3}$ |  |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 2 | 1 | 5 | 2 | 8 | 1 | 3 |  |
| 3 | 2 | 6 | 3 | 9 | - | - |  |
| 4 | - | - | 4 | 12 | - | - |  |

Use Dynamic Programming Problem to obtain the optimal policy for the above problem.
17. Explain the classical static Economic Order Quantity model and derive the expressions for Total Cost per Unit, order quantity, ordering cycle and effective lead time.
18. Explain the important characteristics of a queuing system.

## SECTION C

Answer any TWO questions. Each carries twenty marks.
( $20 \times 2=40$ )
19. Find an optimum integer solution to the following LPP: Mazimize $Z=3 X_{1}+10 X_{2}$, subject to the constraints, $\mathrm{X}_{1}+5 \mathrm{X}_{2} \leq 12, \mathrm{X}_{1} \leq 3$ and $\mathrm{X}_{1}, \mathrm{X}_{2}$ are non-negative integers.
20. Solve the following Non Linear Programming Problem: $\operatorname{Max} Z=-X_{1}^{2}+2 X_{1}+X_{2}$
subject to the constraints, $2 \mathrm{X}_{1}+3 \mathrm{X}_{2} \leq 6 ; 2 \mathrm{X}_{1}+\mathrm{X}_{2} \leq 4$; and $\mathrm{X}_{1}, \mathrm{X}_{2} \geq 0$,
21. Solve the following Quadratic programming Problem, by Wolfe's algorithm.

Max $Z=4 X_{1}+6 X_{2}-2 X_{1} X_{2}-2 X_{1}^{2}-2 X_{2}^{2}$ subject to the constraints,

$$
X_{1}+2 X_{2} \leq 2 ; X_{1}, X_{2} \geq 0 .
$$

22. (i) Consider the economic order quantity with shortage and derive expressions for optimum order quantity.
(ii) For a (M/M/1) : ( $\infty /$ FIFO) queuing model in the steady-state case, derive the steady state difference equations and obtain expressions for the mean and variance of queue length in terms of the parameters $\lambda$ and $\mu$. $(10+10)$

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