SECTION – A

Answer ALL the questions **(10 x 2 = 20 marks)**

1. When do you say that a bounded function is Riemann integrable on [a, b]?
2. Given that X has the probability mass function p(x) = $\frac{1}{8}\ 3_{Cx}$ for x = 0, 1,2, 3 .
   Find the M.G.F. of X.
3. Prove that β(m, n) = β (n,m).
4. Prove that L(Cos at) = $\frac{S}{s^{2+a^2}}$.
5. Evaluate $\int_0^b \int_0^a xy\,(x-y)dx\,dy.$
6. Given f(x,y) = $\begin{matrix} xe^{-x(1+y)}, & x\ \geq 0,\ y\ \geq 0 \\ 0, & other\ wise \end{matrix}$. Find E (X Y).
7. Solve the equation $(D^2+1)y=0.$
8. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} - 4\sqrt{\frac{dy}{dx}}\ = 5.$
9. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find the characteristic equation of A.
10. Show that the system of equations 3x – 4y = 2 ; 5x+2y = 12; - x+3y = 1 are consistent.

SECTION – B

Answer any FIVE questions. **(5 x 8 = 40 marks)**

11. Let f be a bounded function on the closed, bounded interval [a, b] .
    Show that f ∈ R[a,b] if and only if for each ∈>0 there exists a subdivision σ of [a,b] such that U [f ; σ] < L[f ; σ] +∈.....
12. A continuous random variable X has a p.d.f. given by f(x) = $\begin{matrix} k\ x\ e^{-\lambda x}, & x>0,\ \lambda>0 \\ 0, & other\ wise \end{matrix}$.
    Determine the constant k and find the mean and variance of X, when λ is a known constant.
13. Prove that the improper integral $\int_1^\infty \frac{dx}{n}$ diverges.
14. Find L $\left(\frac{1-e^t}{t}\right)$.
15. Given the joint density function of X and Y as f (x, y) = $\begin{matrix} \frac{x}{2}\ e^{-y}, & 0<x<2,\ y>0 \\ 0\ , & other\ wise \end{matrix}$.
    Find the distribution of X+Y.

16. Solve $(D^2 - 2D + 3)\, y = x^3$.

17. Using Laplace transform, solve $\frac{d2y}{dt^2} + \frac{6dy}{dt} + 5y = e^{-2t}$ given that y = 0, $\frac{dy}{dt} = 1$ when t = 0.

18. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$.

SECTION – C

Answer any TWO questions **(2 x 20 = 40 marks)**

19. a) State and prove the first fundamental therorem of integral calculus.

b) The probability density function of the random variable X is $f(x) = \frac{1}{2\theta}\exp\left(-\frac{(x-\theta)}{\theta}\right)$, $-\infty < x < \infty$. Find M.G.F. of X and alsdo find E(X) and var (X).

(10 + 10 = 20)

20. a) Prove that the improper integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ is convergent.

b) Prove that (i) $\left(n + \frac{1}{2}\right) = \frac{1.3.5 \ldots (2n-1)}{2^n}\sqrt{\pi}$ and (ii) (n+1) = n (n).

21. a) Solve the equation $\frac{d2y}{dx^2} - y = (1 + x^2)\, e^x + x\sin x$.

b) The joing p.d.f. of the random variables X and Y is given by f (x,y) = $\begin{matrix} e^{-(x+y)}, & x > 0\ y > 0 \\ 0, & other\ wise \end{matrix}$.

Find the Cov (X,Y) .

22. a) Show that the following system of equations is consistent and hence solve them.

$x - 3y - 8Z = = 10$; $3x + y - 4Z = 0$; $2x + 5y + 6Z = 13$.

b) State and prove Cayley – Hamilton theorem.

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