LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – NOVEMBER 2016

16PST1MC03 / ST 1822 - STATISTICAL MATHEMATICS

Date: 07-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION – A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Define a sequence of real numbers and give an example for it.
2. Write down the formula for s_n of the following sequence and give two subsequences of the sequence 1, 3, 6, 10, 15, ...
3. If L is the limit of the sequence (s_n), then show that every open interval containing L contains all but a finite number of terms of the sequence.
4. Prove that all subsequences of a convergent sequence of real numbers converge to the same limit.
5. State telescoping property of the finite sum. Hence define the telescoping series.
6. Give comparison test for the series of positive terms.
7. Prove that the function $f(x) = x^2$ is unbounded on R but is bounded on each bounded interval of R.
8. Let $f(x) = |x|$ for $x \in (-\infty, \infty)$. Show that 'f' does not have a derivative at '0', even though 'f' is continuous at '0'.
9. Define upper and lower sum of a bounded function 'f' on the closed bounded interval [a, b] with respect to any partition P of [a, b].
10. Define linear independence of k vectors and give an example.

SECTION - B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. Write down the formula for s_n for the sequence 1, -4, 7, -10, 13, ... Find whether or not each of the following sequence is a subsequence of (s_n):
    (i) (1, 7, 13, ...)
    (ii) (-4, -10, -16, ...)
    (iii) (7, -4, 13, -10, ...)
    (iv) (4, 10, 16, ...).
12. For any $a, b \in R$, prove that $\big||a| - |b|\big| \leq |a - b|$. Then prove that ($|s_n|$) converges to $|L|$ if (s_n) converges to L. By an example show that the converse is not true.
13. Show that the limit of the sum of two convergent sequences is the sum of their limits.
14. Let $\sum a_n$ be a series of non-negative numbers and let $s_n = a_1 + a_2 + \ldots + a_n$. Then prove that
    (i) $\sum a_n$ converges if (s_n) is bounded.
    (ii) $\sum a_n$ diverges if (s_n) is not bounded.
15. If $\sum a_n$ converges absolutely, then show that the series $\sum a_n$ converges but not conversely.
16. Show that any series dominated by an absolutely convergent series is convergent.
17. Let $f(x) = x$ $(0 \leq x \leq 1)$. Let σ be the partition $\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ of [0, 1]. Compute U [f; σ] and L[f; σ].
18. State any four properties of the Riemann integral.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19 (a). Prove that a monotonic series converges if and only if it is bounded. **(10)**

19 (b). Prove that the sequence $((1 + \frac{1}{n})^n)$ converges. **(10)**

20. State and prove Leibnitz Rule on alternating series. **(20)**

21(a). Define improper integral of the first, second and third kind and give an example for each kind. **(8)**

21(b). Examine the convergence of the integrals: (i) $\int_1^\infty \frac{1}{x^2}\,dx$ (ii) $\int_0^\infty e^{-x}\,dx$ (iii) $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ **(12)**

22 (a). Give a brief motivation leading to the definition of Taylor series and Maclaurin series.

Hence write the Taylor's formula with integral form of the remainder. **(10)**

22 (b). Describe the inductive procedure of Gram-Schmidt Orthogonalization. **(10)**

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