LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2016**

16UST1MC02/ST 1503 – PROBABILITY AND RANDOM VARIABLES

Date: 07-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Section – A

Answer ALL the questions **(10 * 2 = 20)**

1. What are mutually exclusive events? Give an example.
2. Define mathematical definition of probability.
3. Four cards are drawn at random from a pack of 52 cards. Find the probability that, they are two kings and two queens.
4. Explain conditional probability.
5. Explain mutually independent events.
6. State multiplication law of probability.
7. Show that $P(A \cap B^C) = P(A) - P(A \cap B)$.
8. Define a random variable.
9. Define probability density function.
10. Show that E[a x +b] = a E(x)+b.

Section – B

Answer any FIVE questions **(5 *8 =40)**

11. If A and B are independent events, then show that (a) $\overline{\mathrm{A}}$ and B and (b) $\overline{A}$ and $\overline{B}$ are also independent.
12. An urn contains four tickets marked with numbers 112, 121, 211, 222 and one ticket is drawn at random. Let $A_i\,(i = 1,2,3,...)$ be the event that i^{th} digit of the number of the ticket drawn is 1. Discuss the independence of the events A_1 , A_2 and A_3.
13. Three newspapers A , B and C are published in a certain city, it is estimated from a survey that of the adult population: 20% read A, 16% Read B, 14% read C, * % read A and B, 5% read A and C, 2% read all three. Find what percentage read at least one of the papers?
14. From a city population, the probability of selecting (i) a male or a smoker is 7/10, (ii) a male smoker is 2/5, and (iii) a male, if a smoker is already selected is 2/3. Find the probability of selecting (a) a non-smoker, (b) a male and (c) a smoker, if a male is first selected.
15. State and prove Multiplication law of probability for ‘n’ events.
16. Write the properties of distribution function.
17. A coin is tossed until a head appears. What is the expectation of the number of tosses required?
18. In four tosses of a coin. Let X be the number of heads. Tabulate the 16 possible outcomes with the corresponding values of X,. By simple counting, derive the probability distribution of X and hence calculate the expected value of X.

Section – B

Answer any TWO questions **(2 *20 =40)**

19. (a) A bag contains 17 counters marked with the numbers 1 to 17. A counter is drawn and replaced; a second drawing is then made. What is the probability that: (i) the first number drawn is even and the second odd? (ii) the first number is odd and the second is even ?

    (b) A speaks truth 4 out of 5 times. A die is tossed. He reports that there is a six. What is the chance that actually there was six?

20. (a) For two events A and B ; $P(A)=\frac{3}{4}$; $P(B)=\frac{5}{8}$. Show that (i) $P(A\cup B)\geq\frac{3}{4}$;

    (ii) $\frac{3}{8}\leq P(A\cap B)\leq\frac{5}{8}$ (iii) $P(A\cap B)\leq P(A)\leq P(A\cup B)\leq P(A)+P(B)$.

    (b) A problem in Statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2},\frac{3}{4} and \frac{1}{4}$ respectively. What is the probability that the problem is solved?

21. (a) A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball each other.

    (b) State and prove Baye's theorem.

22. (a) Obtain the mean and variance of the random variable X with pdf *f(x) =*$\lambda\, e^{-\lambda x}$, $0<x<\infty$ zero Otherwise.

    (b) A random X has the following probability function:

Value of X, x	0	1	2	3	4	5	6	7
$p(x)$	0	k	2k	2k	3k	K^2	2k^2	7k^2+k

(i) Find k, (ii) Evaluate $P(X<6), P(X\geq 6), and\ P$ $(iii) P(X\leq a)>\frac{1}{2}$, find the minimum value of a, and (iv) Determine the distribution function of X.

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