LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **NOVEMBER 2016**

ST 2502/ST 2501 – STATISTICAL MATHEMATICS - I

Date: 12-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

PART –A

Answer ALL questions. (10* 2= 20 Marks)

1. Define convergent sequence.
2. Define random variables. Give suitable example.
3. Define absolute convergence and conditional convergence for a series of real numbers.
4. Distinguish between a discrete and a continuous random variable.
5. State mean value theorem.
6. Find the Taylor series for $f(x) = e^x$ about $x = 0$.
7. Verify whether the vectors $\{ (40,15), (-50,25) \}$ are linearly dependent.
8. Define orthogonal vectors.
9. Find the rank of matrix $X = \begin{bmatrix} 1 & 2 & 4 & 4 \\ 3 & 4 & 8 & 0 \end{bmatrix}$.
10. What is a stochastic matrix?

PART –B

Answer any FIVE questions. (5* 8= 40 Marks)

11. Show that every monotonically increasing sequence which is founded above and converges to its least upper bound.
12. Explain that every convergent sequence is bounded. Is the converse is true? Validate your answer.
13. A random variable X takes the value 1,2,3,… and $P(X = x)\frac{1}{2^n}, n = 1,2,3,...$ Find (i) $P(X \ is \ odd)$, (ii) $P(X \le 5)$, (iii) $P(X \text{ is divisible by } 5)$.
14. Let X be a random variable with moment generating function $M_x(t) = \frac{1}{2}(1 + \exp(t))$ Derive the variance of X.
15. Let $P(y_1, y_2, y_3) = \dfrac{y_1 \ y_2 \ y_3}{y_1 + y_2 + y_3}$ Calculate $\dfrac{\partial p}{\partial y_3}(y_1, y_2, y_3)$ at the point $(y_1, y_2, y_3) = (1, -2, 4)$.
16. Find the rank of matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & -1 \end{bmatrix}$
17. Find the inverse of the following matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$.

18. For the joint distribution $f(x,y)=\begin{cases}\frac{9}{4}-x-y, & 0\le x\le x\le 2\ ,0\le y\le 2\\ 0 \text{ otherwise}\end{cases}$.

Obtain the marginal and conditional distributions.

PART –C

Answer any TWO questions. (2* 20= 40 Marks)

19. (a) Define the following (i) Countable set (ii) least upper bound, (iii) greatest lower bound, and (iv) one limit point.
(b) Show that every Cauchy sequence is convergent.

20. (a) State and prove Rolle's Theorem.

(b) Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function. $f(x)=x^3+2x^2-x\ on[-1.2]$

21. (i) Give the joint pdf of (X,Y) as $f(x,y)=\begin{cases}\frac{K}{(1+x+y)^2} \text{ for x > 0, y > 0}\\ 0\ \textit{Otherwise}\end{cases}$. Find K ,

the conditional pdf of X given $Y=y$, Also find $P(X<Y)$.

(ii) State and prove Maclaurin's theorem.

22. (a) Determine the row-rank of $A=\begin{bmatrix}1 & 2 & 1\\ 2 & 3 & 1\\ 1 & 1 & 2\end{bmatrix}$

(b) Explain symmetric matrix and skew symmetric matrix. Give suitable example.

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