LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – NOVEMBER 2016

ST 2962 - MODERN PROBABILITY THEORY

Date: 14-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION A

Answer ALL of the following. **10X2=20**

1. Let ζ be the class of all intervals of the form (x, ∞), x Ɛ R, considered as subset of the real line. Prove ζ is closed under finite unions and finite intersection, but not under complementation.
2. Define: σ – Algebra.
3. If X~U[a, b], then prove that the probability that X lies in the sub interval of (a,b) is proportional to its length.
4. Define: Conditional Probability Space.
5. Define Mixture of Distributions.
6. Explain: Moment Generating functions.
7. Derive the Mean of Beta Distribution of first kind.
8. If $X_n \xrightarrow{p} X, and, X_n \xrightarrow{p} X'$, then prove that X and X' are equivalent.
9. Define: Convergence Weakly.
10. State the three conditions of WLLN.

SECTION B

Answer any FIVE from the following **5X8=40**

11. Let ξ_i be the class of all intervals of the form (a, b), (a<b) a,b Ɛ R, but arbitrary. Then P.T. σ (ξ_i) =Ɓ.
12. Prove that a Field is closed under finite unions. Conversely, a class closed under complementation and finite union is a field.
13. Explain: Induced Probability Space with an example.
14. Prove that Poisson distribution is a limiting case of binomial distribution.
15. State and prove the properties of Expectation of non-negative and arbitrary random variables.
16. State and prove the necessary and sufficient conditions for convergence in probability.
17. Prove that $X_n \xrightarrow{r} X$ => $X_n \xrightarrow{p} X$. If the X_n's are a.s. bounded, conversely $X_n \xrightarrow{p} X$ => $X_n \xrightarrow{r} X$ for all r.
18. State and prove the Kolmogorov SLLN for i.i.d. case.

SECTION C

Answer the following **2x20=40**

19. i)Prove that the intersection of arbitrary number of fields is a field.

(8)

ii) Prove that the distribution function F_X of r.v. X is non-decreasing, continuous on the right with $F_X(-\infty) = 0$ and $F_X(+\infty) = 1$. Conversely, every function F with the above properties is the d.f. of a r.v. on some probability space. **(12)**

20.i) Let $X_n \xrightarrow{p} X, Y_n \xrightarrow{p} Y$, then P.T.

a. $aX_n \xrightarrow{p} aX$ (a, real number)

b. $X_n + Y_n \xrightarrow{p} X + Y$

c. $X_n Y_n \xrightarrow{p} XY$

d. $X_n / Y_n \xrightarrow{p} X / Y$ if P[Yn=0]=0 , for every n, and P[Y=0]=0. **(12)**

ii) Prove that $X_n \xrightarrow{p} X$ implies that $F(X_n) \longrightarrow 0$ for x<c, $F(X_n) \longrightarrow 1$ for $x \geq c$ and conversely.

(8)

21. i) State and Prove Markov's Theorem. **(10)**

ii) Prove that $\sum\sigma^2_n < \infty \Rightarrow \sum(X_n-E(X))$ converges a.s. If X_n's are a.s bounded, converse is also true and we have, $\sum\sigma^2_n < \infty \Leftrightarrow \sum(X_n-E(X))$ converges a.s. **(10)**

22. i) Discuss in detail, the applications of Central Limit Theorem. **(12)**

ii)Let $\{x_n\}$ be a sequence of i.i.d. r.v.'s with characteristic function φ(u). Then prove that

$S_n/n \xrightarrow{p} E(X)$ **(8)**

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