



# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHS, PHYSICS, PLA. BIO. & PLA. BIOTECH., ADV. ZOO.

FOURTH SEMESTER – NOVEMBER 2016

**ST 4201 / ST 4206 / ST 4209 - MATHEMATICAL STATISTICS**

Date: 11-11-2016  
Time: 01:00-04:00

Dept. No.

Max. : 100 Marks

## Section – A

Answer all the questions

10 X 2 = 20 Marks

1. Define independence for three events.
2. Write the sample space for casting three fair coins.
3. Provide any two properties of distribution function.
4. If  $X$  has the p.d.f.  $f(x) = q^x p$ ,  $x = 0, 1, 2, \dots$ ,  $p + q = 1$ , zero elsewhere, find  $E(X)$ .
5. If  $P(A^c) = 2/3$  and  $P(B) = 1/4$  and  $P(A \cap B) = 1/6$ , compute  $P(A^c | B^c)$ .
6. Define standard normal distribution.
7. State the addition theorem on probability for three events.
8. Define the correlation coefficient and state its limits.
9. When an estimator is said to be consistent?
10. Define simple and composite hypothesis.

## Section – B

Answer any five questions

5 X 8 = 40 marks

11. State and prove Boole's inequality.
12. Show that binomial distribution tends to Poisson under certain conditions.
13. Derive mean and variance of binomial distribution.
14. Derive the M.G.F. of normal distribution.
15. Let  $Y_1 < Y_2 < Y_3$  be the order statistics of a random sample of size  $n = 3$  from the exponential distribution with p.d.f.  $f(x) = e^{-x}$ ,  $0 < x < \infty$ , zero elsewhere. Find  $P(Y_3 < 2)$ .
16. Let  $X$  have the p.d.f.  $f(x) = x e^{-x}$ ,  $0 < x < \infty$ , zero elsewhere. Find the moment generating function and hence find mean and variance.
17. If  $X$  and  $Y$  have the joint p.d.f.  $f(x, y) = 2$ ,  $0 < x \leq y < 1$ , zero elsewhere, compute the correlation coefficient between  $X$  and  $Y$ .
18. (a) Write a note on maximum likelihood estimation. **(3 marks)**  
(b) If  $X_1, X_2, \dots, X_n$  is a random sample from Poisson distribution with mean  $\theta$ , find the maximum likelihood estimator of  $\theta$ . **(5 marks)**

**Section-C**

**Answer any two questions**

**2 X 20 = 40 marks**

19(a) State and prove Chebyshev's inequality. **(10 marks)**

(b) State and prove addition theorem on probability for 'n' events. **(10 marks)**

20. (a) Show that for normal distribution mean deviation is  $(4/5)\sigma$ . **(8 marks)**

(b) If X is normal with mean 6 and variance 25, find

(i)  $P(6 < X < 12)$  (ii)  $P(0 < X < 8)$  (iii)  $P(|X - 6| < 10)$ . **(12 marks)**

21.(a) Explain the computation of conditional mean and variance of continuous random variables.

**(5 marks.)**

(b) Let  $f(x,y) = e^{-x-y}$ ,  $0 < x < \infty$ ,  $0 < y < \infty$ , zero elsewhere be the joint p.d.f. of X and Y.

(i) Show that X and Y are independent.

(ii) Compute  $P(X < Y)$ ,  $P(X > 1, Y > 1)$ ,  $P(X = Y)$ ,  $P(X < 2)$  and

$P(0 < X < \infty, X/3 < Y < 3X)$ . **(15 marks)**

22.(a) Derive the probability density function of F distribution. **(15 marks)**

(b) Write any five properties of normal distribution. **(5marks)**

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