LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034
M.Sc. DEGREE EXAMINATION - STATISTICS

FOURTH SEMESTER - NOVEMBER 2016
ST 4814 - ADVANCED OPERATIONS RESEARCH

Date: 10-11-2016
Time: 01:00-04:00 $\square$ Max. : 100 Marks

## SECTION A

Answer ALL questions. Each carries two marks.
( $10 \times 2=20$ )

1. Define a General Linear Programming Problem.
2. What is an unbounded solution in an LPP?
3. What are the applications of goal programming?
4. State the principal of optimality in dynamic programming.
5. What are the three types of integer programming?
6. Define Non Linear Programming Problem?
7. Define a quadratic programming problem.
8. What is inventory control?
9. What is set up cost?
10. What is a queuing system?

## SECTION B

Answer any FIVE questions. Each carries eight marks.
11. Use two-phase simplex method to maximize $Z=3 X_{1}+2 X_{2}$,
subject to the constraints, $2 \mathrm{X}_{1}+1 \mathrm{X}_{2} \leq 1$;

$$
\begin{aligned}
& 3 \mathrm{X}_{1}+4 \mathrm{X}_{2} \geq 12 \\
& \text { and } \mathrm{X}_{1}, \mathrm{X}_{2} \geq 0
\end{aligned}
$$

12. Derive Gomory's constraint for solving a Pure Integer Programming Problem.
13. The relationship between sales S and the amounts $\mathrm{x} \& \mathrm{y}$ spent on 2 advertising media is given by $\mathrm{S}=$ $\frac{200 x}{5+x}+\frac{100 y}{10+y}$. The net profit is $\frac{1}{5}$ of the sales minus the cost of advertising. The advertising budget is maximum 20. Determine how it should be allocated between the 2 media to maximize the net profit. Formulate the above problem and write down all the Khun - Tucker Necessary Conditions.
14. Test for extreme values of $f\left(x_{1}, x_{2}, x_{3}\right)=x_{1}^{2}+x_{2}^{2}+x_{3}^{2}$, subject to the constraints, $x_{1}+x_{2}+3 x_{3}=2$ and $5 x_{1}+2 x_{2}+x_{3}=5$.
15. Use dynamic programming to solve the following problem:

Minimize $\mathrm{z}=y_{1}^{2}+y_{2}^{2}+y_{3}^{2}$
subject to the constraints: $y_{1}+y_{2}+y_{3}=10$ and

$$
\mathrm{y}_{1}, \mathrm{y}_{2}, \mathrm{y}_{3} \geq 0
$$

16. A corporation is entertaining proposals from its 3 plants for possible expansion of its facilities. The corporation's budget is $£ 5$ millions for allocation to all 3 plants. Each plant is requested to submit its proposals giving total cost C and total revenue R for each proposal. The following table summarizes the cost and revenue in millions of pounds. The zero cost proposals are introduced to allow for the probability of not allocating funds to individual plants. The goal of the corporation is to maximize the total revenue resulting from the allocation of $£ 5$ millions to the three plants.

|  | Plant 1 |  | Plant 2 |  | Plant 3 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Proposal | $\mathrm{C}_{1}$ | $\mathrm{R}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{R}_{2}$ | $\mathrm{C}_{3}$ | $\mathrm{R}_{3}$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 5 | 2 | 8 | 1 | 3 |
| 3 | 2 | 6 | 3 | 9 | - | - |
| 4 | - | - | 4 | 12 | - | - |

Use Dynamic Programming Problem to obtain the optimal policy for the above problem.
17. Explain the classical static Economic Order Quantity model and derive the expressions for Total Cost per Unit, order quantity, ordering cycle and effective lead time.
18. Explain the elements of a queuing system.

## SECTION C

Answer any TWO questions. Each carries twenty marks.
19. Find an optimum integer solution to the following LPP:

Mazimize $Z=7 X_{1}+9 X_{2}$, subject to the constraints,
$-\mathrm{X}_{1}+3 \mathrm{X}_{2} \leq 6,7 \mathrm{X}_{1}+\mathrm{X}_{2} \leq 35$ and $\mathrm{X}_{1}, \mathrm{X}_{2}$ are non-negative integers.
20. Solve the following Non Linear Programming Problem:

$$
\operatorname{Max} Z=7 X_{1}^{2}+6 X_{1}+5 X_{2}^{2}
$$

subject to the constraints, $X_{1}+2 X_{2} \leq 10 ; X_{1}-3 X_{2} \leq 9$; and $X_{1}, X_{2} \geq 0$,
21. Solve the following Quadratic programming Problem, by Wolfe's algorithm.

Max $Z=4 X_{1}+6 X_{2}-2 X_{1} X_{2}-2 X_{1}^{2}-2 X_{2}^{2}$ subject to the constraints,

$$
\mathrm{X}_{1}+2 \mathrm{X}_{2} \leq 2 ; \mathrm{X}_{1}, \mathrm{X}_{2} \geq 0 .
$$

22. (i) Consider the economic order quantity with one price break and derive expressions for optimum order quantity and Total cost per unit.
(ii) For a (M/M/1) : ( $\infty /$ FIFO) queuing model in the steady-state case, derive the steady state difference equations and obtain expressions for the mean and variance of queue length in terms of the parameters $\lambda$ and $\mu$.
$(10+10)$
