LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – NOVEMBER 2017

17/16PST1MC01/ST1815/ST1820 – ADVANCED DISTRIBUTION THEORY

Date: 02-11-2017 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

PART – A

Answer all the questions **(10 x 2=20)**

1) Give any two equations that can be used to verify the independence of two random variables X and Y.
2) Write the pdf of Binomial distribution truncated at 0.
3) Show that Geometric distribution satisfies Lack of memory property.
4) Let (X, Y) have Bivariate Binomial $BB(n, p_1, p_2, p_{12})$. Obtain the marginal distribution of X .
5) Obtain the mean of log normal distribution.
6) Write the joint pdf of i^{th} and j^{th} order statistics.
7) Explain Compound distributions.
8) Define non-central F variable.
9) Let X_1, X_2, X_3 be independent Normal variables with $E(X_1) = 2, E(X_2) = 3, E(X_3) = 4,$ $V(X_1) = 2, V(X_2) = 2, V(X_3) = 3$. Check whether $X_1 + X_2 - 2X_3$ and $X_1 - X_2$ are independent.
10) Define a Quadratic form in n variables.

PART – B

Answer any 5 questions **(5 x 8=40)**

11) Show that Lack of memory property characteristics the Exponential distribution when the random variable is non negative and continuous.
12) Let X_1 and X_2 be i.i.d geometric random variables. Obtain the Conditional distribution of X_1 given $X_1 + X_2 = n.$
13) Obtain $E(X / Y = y)$ for a Bivariate Normal distribution. Show that when $° = 0,$ the random variables X and Y are independent for Bivariate normal distribution..
14) Show that $E[E(X / Y)] = E(X)$ and $V[X] = E[V(X / Y)] + V[E(X / Y)].$
15) Derive the pdf of k^{th} Order statistic.
16) Let $F(x) = \begin{cases} 0 & ; \ x < 0 \\ \dfrac{x+1}{2} & ; \ 0 \le x < 1 \\ 1 & ; \ 1 \le x \end{cases}$ be the distribution function of a random variable. Obtain the decomposition of F, MGF.
17) Let X and Y be independent Gamma random variables, X with $G(\sim_1, p_1)$ and Y with $G(\sim_2, p_2)$. Obtain the distribution of $U = \dfrac{X}{X+Y}$.
18) Derive the MGF of trinomial distribution. Obtain $Cov(X_1, X_2)$.

PART- C

Answer any 2 questions **(2 x 20=40)**

19) a) Obtain the PGF of a Bivariate Poisson distribution.

b) Obtain (/ =) and (/ =). Hence obtain the correlation coefficient between X_1 and X_2.

C) Show that $a_{12} = 0 \Leftrightarrow$ independence of X_1 and X_2. (5+10+5 marks)

20) a) Derive the pdf of Non central t-distribution.

b) Let X have Binomial (n, p) for a fixed n. Let n be a random variable having Poisson distribution with parameter a. Obtain the unconditional distribution of X. (15+5 marks)

21) a) Let X_1 and X_2 be i.i.d random variables with finite second moment. Show that X_1 is Normal iff $X_1 + X_2 \amalg X_1 - X_2$. (10+5+5)

b) Let X_i have $N(«_i, ±_i^2)$ $\forall i = 1,2,....,n$. Let X_i be independent. Show that $\sum a_i x_i \amalg \sum b_i x_i$ if and only if $\sum a_i b_i ±_i^2 = 0$.

c) Give an example to show that the random variables X and Y are marginally Normal, but jointly not Normal.

22) a) Let A_n be an increasing sequence of events. Show that $p(\lim A_n) = \lim p(A_n)$. Deduce the result for decreasing sequence of events. (10+5+5)

b) Show that $F(x)$ is right continuous.

c) Consider a Box containing M white balls and $N - M$ red balls. Let n balls be drawn

1) With replacement 2) Without replacement.

Obtain the probability distribution of the number of white balls drawn in both the cases.

$$$$$$$$