LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **STATISTICS**

FIRSTSEMESTER – NOVEMBER 2017

17/16PST1MC03 /ST 1822- STATISTICAL MATHEMATICS

Date: 08-11-2017　　Dept. No. 　　Max. : 100 Marks
Time: 01:00-04:00

SECTION – A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Let A = {x │ x = $\frac{4n+3}{n}$, n ϵ N}. Find the lub and glb of A.
2. Give an example for the following functions: (i) f is onto (ii) f is many one into.
3. State any two properties of the Riemann integral.
4. Check whether or not the sequence (log$\frac{1}{n}$) diverges to minus infinity.
5. If (s_n) converges to 'L', then show that ($(-1)^n s_n$) oscillates.
6. Write the formula for differentiating the sum, difference, product and quotient of two functions.
7. Show that f(x) = [x] on [1, ∞] is monotonic increasing but not strictly increasing.
8. Give an example for a function which is continuous at a point but not differentiable at that point.
9. Define upper sum and lower sum of a bounded function 'f' defined on the closed bounded interval [a, b] with respect to any partition P of [a, b].
10. Distinguish between linear independence and linear dependence of k vectors and give an example.

SECTION - B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. If $\sum a_n$ is a series of non-negative numbers and $s_n = a_1 + a_2 + \ldots + a_n$, then show that

    (i) $\sum a_n$converges if (s_n) is bounded.

    (ii) $\sum a_n$diverges if (s_n) is not bounded.

12. If $\sum a_n$ converges absolutely, then prove that the series $\sum a_n$ converges but not conversely.
13. Let f(x) = 1 + x when x > 1, f(x) = 1 – x when x < 1 and f(x) = 0 at x = 1. Find $\lim_{x\to 1} f(x)$.
14. Examine the continuity of the function at the origin:

$$f(x) = \begin{cases} \frac{x - |x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

15. Give a brief motivation leading to the definition of Taylor series and Maclaurin series expansion of a function f.

16. Define improper integrals of first, second, and third kind and give an example for each kind.

17. Let f(x) = x ($0 \leq x \leq 1$). Let P be the partition $\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ of [0, 1]. Compute U [f; P] and L[f; P].

18. If a sequence of real numbers (s_n) converges to 'L', then prove that any subsequence of (s_n) also converges to 'L'.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19(a) State and prove a necessary and sufficient condition for the convergence of a monotonic sequence. (10)

19(b) Examine the convergence of the sequence $((1 + \frac{1}{n})^n)$. (10)

20(a) State and prove Leibnitz fundamental theorem on alternating series. (15)

20(b) Illustrate that if any one of the two conditions for convergence is removed from the Leibnitz theorem, then the series need not be convergent. (05)

21(a) Define the convergence of the improper integrals of first kind and examine the convergence of the integrals: (i) $\int_1^\infty \frac{1}{x^2}\, dx$ (ii) $\int_0^\infty e^{-x}\, dx$ (iii) $\int_1^\infty \frac{1}{\sqrt{x}} dx$ (12)

21(b) Use μ - Test for testing the convergence of (i) $\int_0^\infty \frac{x^2 dx}{(k^2 + x^2)^2}$ (ii) $\int_0^\infty \frac{x^3 dx}{(k^2 + x^2)^2}$. (08)

22(a) Discuss in detail about the inductive procedure of Gram-Schmidt Orthogonalization. (10)

22(b) Write an explanatory note on the characteristic value problem and give the definition of the characteristic roots and vectors of a matrix with an illustration. (10)

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