LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2017**

ST 3506 – MATRIX AND LINEAR ALGEBRA

Date: 07-11-2017 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL the questions: **(10 x 2 = 20 marks)**

1. Define symmetric matrix.
2. Define transpose of a matrix with an example.
3. Define cofactor of an element in a matrix and hence give example.
4. Find $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$.
5. Find inverse of the matrix A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$
6. Define linear independence of vectors.
7. Find the rank of the matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$
8. Define 'Basis' of a vector space.
9. Write any two properties of linear transformation.
10. Define characteristic vector of a matrix

PART – B

Answer any FIVE questions: **(5 x 8 = 40 marks)**

11. Show that every square matrix A can be expressed uniquely as P + I Q where P and Q are Hermitian matrices.
12. Evaluate the determinant $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$
13. Show that Adj (A^T) = (Adj A)^T
14. Using Cramer's rule find the solution of:
    3 x + 5 y – 7 z = 13
    4x + y – 12 z = 6
    2x + 9 y – 3 z = 20
15. Show that no skew-symmetric matrix can be of rank 1.
16. Solve the following system of equations by matrix inversion method:
    3 x – 2 y + 3 z = 8
    2 x + y – z = 1
    4x – 3 y + 2 z = 4
17. State and prove Cayley-Hamilton Theorem.

18. Use Laplace method of expansion to show that

$$\begin{vmatrix} a & -b & -a & b \\ b & a & -b & -a \\ c & -d & c & -d \\ d & c & d & c \end{vmatrix} = 4.(a^2+b^2).(c^2+d^2)$$

PART – C

Answer any TWO questions: **(2 x 20 = 40 marks)**

19. (a) If A and B commute, obtain $(A + B)^n$

(b) If A is a Hermitian matrix, show that iA is a skew-Hermitian matrix. **(10 + 10)**

20. (a) If A and B are square matrices of the same order, prove that Adj (AB) = (Adj B). (Adj A)

(b) Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$ **(10 + 10)**

21. (a) Find the inverse of the matrix

$$\begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 2 & 1 \\ 3 & 2 & 1 & 4 \end{bmatrix}$$

(b) Find the rank of the matrix

$$\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$

(12 + 8)

22. (a) Find the characteristic equation of the following matrix and hence find its inverse using Cayley-Hamilton Theorem:

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & -3 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

(b) Verify whether the following three vectors are linearly independent:

$$\begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

(12 + 8)

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