LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – **NOVEMBER 2018**

16UST5MC01– APPLIED STOCHASTIC PROCESSES

Date: 27-10-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

SECTION A (10 X 2 = 20 MARKS)
Answer ALL the questions

1. Define Stochastic process. Give an example.

2. In the fair coin experiment, we define the process {X(t)} as follows

$$X\ (t) = \begin{cases} \sin \pi t, & if\ head\ \ shows\ and \\ 2t, & if\ tail\ shows \end{cases}$$

Find E{X(t)}

3. Define Markov Chain.

4. What is Transition Probability Matrix?

5. Define periodicity of a markov chain.

6. Frame the two state Markov chain tpm : The probability of a dry day (state 0) following a rainy day (state 1) is 1/3 and that the probability of a rainy day following a dry day is 1/2 .

7. Define Transient state.

8. When do we say that a Markov chain is Ergodic?

9. List out any 2 properties of Poisson process.

10. Explain Branching process.

SECTION – B (5 X 8 = 40 MARKS)
Answer any FIVE questions :

11. If a state is recurrent show that any state communicating with it is also recurrent.

12. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes (i) exactly 4 customers arrive and (ii) more than 4 customers arrive.

13. Verify whether the following markoc chain is recurrent, aperiodic and irreducible.

$$p = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1/2 & 1/8 & 1/8 & 1/4 \end{pmatrix}$$

14. State and prove the additive property of Poisson process

15. Explain the birth and death process and obtain the forward differential equations.

16. State and prove Chapman-kolmogrov equation for a markov chain.

17. check whether the process is stationary or evolutionary $X(t) = A_1+A_2t$, where A_1 and A_2 are independent random variables with $E(A_i) = a_i$ $V(A_i) = \sigma_i^2$, I = 1,2

18. Define (i) absorbing state (ii) second order process (iii) application of Basic limit theorem.

SECTION – C (2 X 20 = 40 MARKS)
Answer any TWO questions

19. Derive the expression $P_n(t)$ for poisson process.

20.Let $\{X_n, n>0\}$ be a Markov chain having state space S = {1,2,3,4} and transition probability matrix

$$P = \begin{pmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}$$

Verify whether the states are transient or recurrent. Obtain the equivalence classes. Find the period of the states.

21. Explain the classification of the stochastic process based on time and state space with examples.

22. What is meant by extinction of a process? With usual notation prove that the probability of ultimate extinction is 1, if m<1.

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