## LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034

M.Sc. DEGREE EXAMINATION - STATISTICS

THIRD SEMESTER - NOVEMBER 2019

## 17/18PST3ES01 - ADVANCED OPERATIONS RESEARCH

Date: 06-11-2019
Dept. No. $\square$ Max. : 100 Marks
Time: 09:00-12:00

## SECTION A

Answer ALL questions. Each carries two marks.

1. Define a General Linear Programming Problem.
2. What is an optimum solution of a linear programming problem?
3. What are the applications of dynamic programming?
4. State the principal of optimality in dynamic programming.
5. What is integer programming?
6. Define Non Linear Programming Problem?
7. Define a quadratic programming problem.
8. What is inventory control?
9. What are the costs associated with inventory?

10 . What is queuing theory?

## SECTION B

Answer any FIVE questions. Each carries eight marks.
11. Use two-phase simplex method to maximize $Z=5 X_{1}+8 X_{2}$, subject to the constraints, $3 X_{1}+2 X_{2} \geq 3$; $X_{1}+4 X_{2} \geq 4 ; X_{1}+X_{2} \leq 5$; and $X_{1}, X_{2} \geq 0$.
12. Derive Gomory's constraint for solving a Mixed Integer Programming Problem.
13. State the necessary conditions for solving the following Quadratic programming Problem. Max $\mathrm{Z}=6 \mathrm{X}_{1}+$ $3 X_{2}-4 X_{1} X_{2}-2 X_{1}^{2}-3 X_{2}^{2}$ subject to the constraints, $X_{1}+X_{2} \leq 1 ; 2 X_{1}+3 X_{2} \leq 4$; and $X_{1}, X_{2} \geq 0$, and show that Z is strictly concave.
14. Test for extreme values of $f\left(x_{1}, x_{2}, x_{3}\right)=x_{1}^{2}+x_{2}^{2}+x_{3}^{2}$, subject to the constraints, $x_{1}+x_{2}+3 x_{3}=2$ and $5 x_{1}+2 x_{2}+x_{3}=5$.
15. Using Dynamic Programming Problem, maximize $\mathrm{z}=\left\{\mathrm{y}_{1} \cdot \mathrm{y}_{2} \ldots . . \mathrm{y}_{\mathrm{n}}\right\}$ subject to the constraints, $\mathrm{y}_{1}+\mathrm{y}_{2}+\ldots . .+$ $\mathrm{y}_{\mathrm{n}}=\mathrm{c}$, and $\mathrm{y}_{\mathrm{j}} \geq 0$.
16. A corporation is entertaining proposals from its 3 plants for possible expansion of its facilities. The corporation's budget is $£ 5$ millions for allocation to all 3 plants. Each plant is requested to submit its proposals giving total cost C and total revenue R for each proposal. The following table summarizes the cost and revenue in millions of pounds. The zero cost proposals are introduced to allow for the probability of not allocating
funds to individual plants. The goal of the corporation is to maximize the total revenue resulting from the allocation of $£ 5$ millions to the three plants.

|  | Plant 1 |  | Plant 2 |  | Plant 3 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Proposal | C | $\mathrm{R}_{1}$ | C | $\mathrm{R}_{2}$ | C | $\mathrm{R}_{3}$ |
|  | 1 |  | 2 |  | 3 |  |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 5 | 2 | 8 | 1 | 3 |
| 3 | 2 | 6 | 3 | 9 | - | - |
| 4 | - | - | 4 | 12 | - | - |

Use Dynamic Programming Problem to obtain the optimal policy for the above problem.
17. Explain the classical static Economic Order Quantity model and derive the expressions for Total Cost per Unit, order quantity, ordering cycle and effective lead time.
18. Explain the important characteristics of a queuing system.

## SECTION C

Answer any TWO questions. Each carries twenty marks.
19. Find an optimum integer solution to the following LPP: Mazimize $Z=3 X_{1}+10 X_{2}$, subject to the constraints, $\mathrm{X}_{1}+5 \mathrm{X}_{2} \leq 12, \mathrm{X}_{1} \leq 3$ and $\mathrm{X}_{1}, \mathrm{X}_{2}$ are non-negative integers.
20. Solve the following Non Linear Programming Problem: Max $Z=7 X_{1}{ }^{2}+6 X_{1}+5 X_{2}{ }^{2}$ subject to the constraints, $\mathrm{X}_{1}+2 \mathrm{X}_{2} \leq 10 ; \mathrm{X}_{1}-3 \mathrm{X}_{2} \leq 9 ;$ and $\mathrm{X}_{1}, \mathrm{X}_{2} \geq 0$,
21. Solve the following Quadratic programming Problem, by Wolfe's algorithm.

Max $Z=4 X_{1}+6 X_{2}-2 X_{1} X_{2}-2 X_{1}{ }^{2}-2 X_{2}^{2}$ subject to the constraints,

$$
\mathrm{X}_{1}+2 \mathrm{X}_{2} \leq 2 ; \mathrm{X}_{1}, \mathrm{X}_{2} \geq 0
$$

22. (i) Consider the economic order quantity with price breaks and derive expressions for optimum order quantity and Total cost per unit.
(ii) For a (M/M/1) : ( $\propto$ /FIFO) queuing model in the steady-state case, derive the steady state difference equations and obtain expressions for the mean and variance of queue length in terms of the parameters $\lambda$ and
