LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2019**

18PST1MC03 – STATISTICAL MATHEMATICS

Date: 05-11-2019 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION – A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Write down the formula for s_n of the following sequence and give a subsequence of the sequence 1, 3, 6, 10, 15, ...
2. Show that the sequence $\left(\frac{1}{n}\right)$ has the limit L = 0.
3. If (s_n) is a sequence of real numbers and if $\lim_{n\to\infty} s_{2n} = L$ and $\lim_{n\to\infty} s_{2n-1} = L$, then prove that $s_n \to L \text{ as } n \to \infty$.
4. Prove that the sequence $\left(\log\frac{1}{n}\right)$ diverges to minus infinity.
5. Test whether the series $\sum \frac{1+n}{1+3n}$ is convergent or not.
6. State comparison test for the series of positive terms.
7. If f is bounded on A and g is unbounded on A, then prove that f + g is unbounded on A.
8. Give the different formulae for differentiating the sum, and product of two functions f and g which are both differentiable at x = a in R.
9. Define upper and lower integral of a bounded function ‘f’ on the closed and bounded interval [a, b].
10. Define linear independence or linear dependence of k vectors and give an example.

SECTION - B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. For any a, b ϵ R, prove that $\big|\,|a| - |b|\,\big| \le |a-b|$. Then prove that $(|s_n|)$ converges to $|L|$ if (s_n) converges to L. Show that the converse is not true.

12. Let $\sum a_n$ be a series of non-negative numbers and let $s_n = a_1 + a_2 + \ldots + a_n$. Prove that

    (i) $\sum a_n$ converges if (s_n) is bounded.

    (ii) $\sum a_n$ diverges if (s_n) is not bounded.

13. If $\sum a_n$ converges absolutely, then prove that the series $\sum a_n$ converges but not conversely.

14. Let f(x) = 1 + x when x > 1, f(x) = 1 – x when x < 1 and f(x) = 0 at x = 1. Find $\lim_{x \to 1+} f(x)$, $\lim_{x \to 1-} f(x)$, and $\lim_{x \to 1} f(x)$.

15. State (i) Extreme-value theorem, (ii) Intermediate value theorem, and (iii) Fixed point theorem for continuous functions on a bounded and closed interval [a, b].

16. State Mean Value Theorem for Derivatives. Show by an example that the conclusion of this theorem may fail to be true if there is any point between a and b where the derivative of the function does not exist.

17. Let f(x) = x ($0 \leq x \leq 1$). Let σ be the partition $\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ of [0, 1]. Find U [f; σ] and L[f; σ].

18. If f ∈ R[a, b] and g ∈ R[a,b] and if f(x) ≤ g(x) almost everywhere on [a, b], then show that $\int_a^b f \leq \int_a^b g$.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19(a). Prove that a monotonic sequence converges if and only if it is bounded. **(10)**

19(b). Prove that the Geometric sequence (x^n) converges to 0 if $0 < x < 1$ and diverges to infinity if $1 < x < \infty$. **(10)**

20. State and prove the fundamental theorem on alternating series (Leibnitz Rule). **(20)**

21(a). If f(x) = $x\,|x|$ for $x \in$ R, then show that f'(x) = $2|x|$ for every x in R. **(8)**

21(b). Examine the convergence of the improper integrals of the first kind:
(i) $\int_1^\infty \frac{1}{x^2}\,dx$ (ii) $\int_0^\infty e^{-x}\,dx$ (iii) $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$. **(12)**

22(a). Give a motivation leading to the definition of Taylor series and Maclaurin series for f. State the Taylor's formula with integral form of the remainder. **(10)**

22(b). Demonstrate the Gram-Schmidt Orthogonalization technique. **(10)**

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